Short Wavelength Temperature Gradient Driven Modes in Tokamak Plasmas

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New unstable temperature gradient driven modes in an inhomogeneous tokamak plasma are identified. These modes represent temperature gradient (ion and electron) driven modes destabilized in the short wavelength regions with $k_{\perp} \rho_{i,e} \gg 1$, respectively. The instability occurs due to a specific plasma response that significantly deviates from Boltzmann distribution in the regions $k_{\perp} \rho_{i,e} \gg 1$.

By using Ampère’s law and the Poisson equation, we obtained a general dispersion equation

$$k_{\perp}^2 \delta^2 (l_1 \tau + l_e + D_e \tau + D_e) - 2s^2 (D_e \tau + D_e) (l_1 \tau + l_e) = -k_{\perp}^2 \lambda_D^2 [k_{\parallel}^2 \delta^2 - 2s^2 (D_e \tau + D_e)]. \tag{2}$$

where $\lambda_D$ is the Debye length, $\lambda_D = D_e/(4 \pi n_0 e^2)$, $\delta^2 = c^2/\omega^2$, and $\tau = T_e/T_i$. This general dispersion equation applies to both standard ITG and ETG modes as well as to new short wavelength modes. A similar dispersion equation was analyzed in Ref. [13].

The dispersion Eq. (2) is solved as a function of the $k_{\rho} \rho_i$ for fixed plasma parameters [13]: $\beta = 2 \times 10^{-4}$, $\rho_i/L_n = 2\sqrt{2} \times 10^{-2}$, $\rho_i/L_T = \sqrt{2} \times 10^{-1}$, $\rho_i/L_r = \sqrt{2} \times 10^{-1}$, $k_{\rho_i} = \sqrt{2} \times 10^{-1}$, $k_{\rho_e} = 2 \sqrt{2} \times 10^{-3}$, $\tau = 1$, and $\lambda_D = 0$. The mode frequency and growth rate normalized to $k_{\parallel} v_{th,e}$ are shown in Figs. 1(a) and 1(b), respectively. Two new unstable branches exist in the regions $k_{\rho} \rho_i \simeq 1$ and $k_{\rho} \rho_e \simeq 1$. Recently, it has been emphasized that the ETG modes can be significantly modified by a finite value of the Debye length parameter $[11,14]$. Numerical solution of (2) shows that the electron short wavelength mode is strongly stabilized in a high temperature plasma for $\lambda_D/\rho_e \simeq 1$; it is completely suppressed for $\lambda_D/\rho_e \simeq 3$.

New modes occur due to a specific plasma response for $k_{\rho} \rho_a > 1$. A standard notion is that for large values of the Larmor radius parameter, $k_{\rho} \rho_a > 1$, the density response of the respective plasma component is Boltzmann due to decaying asymptotics of $\Gamma_{0,1}(b_a) \sim 1/\sqrt{b_a}$ for large $b_a$. This, in fact, implicitly assumes that the ratio $\omega_{na}/\omega$ is finite for large $k_{\rho} \rho_a$. In turn, this requires that the mode eigenfrequency increases with $k_{\rho} \rho_a$ (linearly or faster). The latter is true for drift wave type modes, where $\omega \sim \omega_{na}$; however, temperature gradient driven modes are basically sound waves whose frequency is of the order of the shearless case. If the wave frequency $\omega$ remains approximately constant, the ratio $\omega_{na}/\omega$ will increase with $k_{\rho} \rho_a$ that compensates for the decaying $1/\sqrt{b_a}$ factor from $\Gamma_{0,1}(b_a)$. Then the response functions are simplified for $k_{\rho} \rho_a \gg 1$, giving...
\[ l_\alpha + D_\alpha = 1 + \frac{1}{2} \frac{e_a}{\sqrt{\pi}} \frac{v_{ta}}{\omega L_n} s_a Z(s_a) \left( 1 - \frac{\eta_a}{2} \right) - \frac{1}{2} \frac{v_{ta}}{\omega L_n} s_a \left[ \frac{1}{2} Z(s_a) - s_a^2 Z(s_a) \right], \]

(3)

where \( \eta_a = L_{na}/L_{Ta} \).

For the electron short wavelength branch, the ions can be taken adiabatic. Then in the electrostatic limit and \( \lambda_D = 0 \), \( \tau = 1 \), the dispersion equation reduces to \( 1 + l_e + D_e = 0 \). Solution of this dispersion equation with (3) is shown in Fig. 1 by squares. In the fluid limit \( \omega \gg k_v v_{te} \) plasma dispersion functions can be simplified, giving

\[ 2 + \frac{1}{2} \frac{v_{te}}{\sqrt{\pi} \omega L_n} \left( 1 - \frac{\eta_e}{2} \right) + \frac{1}{4} \frac{v_{te}}{\sqrt{\pi} \omega L_n} \left( 1 + \frac{\eta_e}{2} \right) = 0. \]

(4)

In the leading order one has from this equation \( \omega^3 = v_t^2 k_i^2 (1 + \eta_i/2)/(8 \sqrt{\pi} L_n) \). The ion short wavelength instability exists even for adiabatic electrons; however, the mode growth rate is further increased due to the electron Landau damping reaching the maximum at \( \eta_e \sim 5 \). Effect

For the ion short wavelength branch \( k_i \rho_i \gg 1 \), electron finite Larmor radius is not important, \( k_i \rho_e < 1 \), and electron Landau damping can be taken into account in the first order in \( \omega/k_i v_{te} < 1 \). Then the electron response function in this regime is

\[ l_e + D_e = 1 + is_e \sqrt{\pi} \left( 1 - \frac{\omega_{ne} + \omega_T}{\omega} \right). \]

(5)

Electrostatic limit of (2) with the ion (3) and electron (5) response, and \( \lambda_D = 0 \), gives the ion mode frequency shown in Fig. 1 by circles.

In the fluid limit \( \omega \gg k_i v_{ni} \), one can get the following dispersion equation for the ion mode:

\[ \omega_{ni} \frac{1}{\omega} \left( 1 + \frac{\eta_i}{2} \right) + 2 + \omega_{ni} \frac{1}{\omega} \sqrt{\pi k_i \rho_i} \left( 1 - \frac{\eta_e}{2} \right) - is_e \sqrt{\pi} \omega_{ne} \omega \left( 1 - \frac{\eta_e}{2} \right) = 0. \]

(6)

of the ion temperature gradient on the ion short wavelength mode eigenfrequency is shown in Fig. 2 as a function of the \( k_i \rho_i \) for a different value of \( \eta_i \) [from Eq. (2)]. For \( \eta_i = 3 \),

FIG. 1. Normalized wave frequency (a) and growth rate (b) for ITG (left panel) and ETG (right panel) modes. Solid line: standard ITG and short wavelength ion mode; dotted line: standard ETG and short wavelength electron mode; circle: simplified model \( l_e + D_e + \tau (l_i + D_i) = 0 \) with (3) and (5); square: \( 1 + l_e + D_e = 0 \) with (3).

FIG. 2. The \( k_i \rho_i \) dependence of the normalized real \( \omega_r \) and imaginary \( \omega_i \) parts of the eigenfrequency for ion mode. Solid line: \( \eta_i = 5 \); dotted line: \( \eta_i = 4 \); dotted-dashed line: \( \eta_i = 3 \).
the second peak disappears; however, there is still an instability in the short wavelength region.

The above shearless slab analysis is applicable to tokamak plasmas in the regions of a weak/negative magnetic shear located around the minimum $q$ surface where the parallel wave vector $k_\parallel \neq 0$. Such a situation may occur near the double mode-rational surface [11,12,15] in the reversed shear regimes. Toroidal drift and mode coupling effects are weak in this region and can be neglected.

To investigate how the short wavelength modes may be affected by a finite magnetic shear and magnetic drift effects, we consider a simple nonlocal model based on the differential eigenmode equation. To illustrate the basic destabilization mechanism due to ion dynamics, we assume adiabatic electrons. A standard gyrokineastic equation in the ballooning space [4,8,16,17] gives

$$2\phi(\theta) = \int F_m J_0(k_\perp v_\perp/\omega_c)^2 v \frac{\omega - \omega_*}{\omega - \omega_D + i\nu_1/qR\partial/\partial \theta} \times [J_0(k_\perp v_\perp/\omega_c)\phi(\theta)], \tag{7}$$

where $\omega_* = \omega_{ni} + \omega_{J_1}/(v_i^2/v_i^2 - 3/2)$, $\omega_D = \omega_D(\cos \theta + s\theta \sin \theta)(v_i^2/2v_i^2 + v_i^2/v_i^2)$, and $\theta$ is the ballooning space variable. In the fluid limit $\omega > k_\parallel v_i$ and $\omega > \omega_D$, one can obtain from (7) the following eigenmode equations [18,19]

$$2\phi(\theta) = \left(1 - \frac{\omega_{ni}}{\omega}\right)G_0(b)\phi(\theta) - \frac{\omega_{ri}}{\omega} G_1(\theta)\phi(\theta) \left[1 - \frac{\omega_{ni}}{\omega}\right] \left(1 - \frac{\omega_{ri}}{\omega} \right) G_2(\theta) \phi(\theta) + \frac{v_i^2}{q^2 R^2 \omega^2} \phi(\theta) C_1 \frac{\partial^2 \phi(\theta)}{\partial \theta^2} + C_2 \frac{\partial \phi(\theta)}{\partial \theta} + C_3 \phi(\theta) + \frac{v_i^2}{q^2 R^2 \omega^2} \phi(\theta) \left[D_1 \frac{\partial^2 \phi(\theta)}{\partial \theta^2} + D_2 \frac{\partial \phi(\theta)}{\partial \theta} + D_3 \phi(\theta) \right].$$

Here $\omega_D = 2\nu_i \omega_{ni}$ is the toroidal drift frequency, $\nu_i = L_i/R$ is the toroidicity parameter, $s$ is the shear parameter, and various coefficients are defined as follows: $G_1 = b[G_1(b) - G_0(b)]$, $G_2 = G_0(b) + b\Gamma_1(b) - G_0(b)]/2$, $C_1 = G_0(b)/2$, $D_1 = G_0(b)/2 + G_1/2$, $C_2 = k_\perp^4 \partial G_1/\partial G_1$, $D_2 = -k_\perp^4 \partial G_1/\partial G_4$, $G_4 = 2[bG_1 + G_0(b)] - b\Gamma_1/2]$, $C_3 = -(k_\perp^4 \partial k_\perp/\partial G_2 + k_\perp^4 \partial^2 k_\perp/\partial G_1^2 + k_\perp^4 \partial^2 k_\perp/\partial G_2^2 + k_\perp^4 \partial^2 k_\perp/\partial G_3^2 + k_\perp^4 \partial^2 k_\perp/\partial G_4^2$. $G_2 = G_0(b) + b\Gamma_1(b) - G_0(b)]/2$, $G_3 = G_0(b) + b\Gamma_1(b)/2 - 2\Gamma_1/b) + b^2[G_0(b) - G_1(b)]$, where $b = k_\perp^2 R^2(1 + s^2 \theta^2)/2$.

The long wavelength limit of Eq. (8) has been considered in [4,19]. In this Letter, we consider a short wavelength limit $k_\perp^2 R^2 > 1$, where we obtain the function $F = \phi/\sqrt{k_\perp}$ the following Weber equation:

$$A \frac{\partial^2 F(\theta)}{\partial \theta^2} + \left(a_1 + \frac{3 \omega_D}{4 \omega} a_0 - 2\sqrt{\pi} k_\perp \rho_i \right) F + \theta^2 \left(\frac{3 \omega_D}{4 \omega} a_0 \left(s - \frac{1}{2}\right) - \sqrt{\pi} k_\perp \rho_i s^2 + \frac{v_i^2}{q^2 R^2 \omega^2} a_2 s^4 \frac{k_\perp^2 \rho_i^2}{4} - \frac{3}{4} A s^4 \right) F = 0. \tag{9}$$

Here, $A = -v_i^2/(2q^2 R^2 \omega^2)^{-1} a_0$, $a_0 = 1 - (1 + \eta_2)/2 \omega_{ni}/\omega$, $a_1 = 1 - (1 - \eta_2)\omega_{ni}/\omega$, and $a_2 = 1 - (1 + 3\eta_2)/2 \omega_{ni}/\omega$. We have also used the expansion $\theta^2 < 1$. The last two terms in Eq. (9) are due to the $C_2$, $C_3$, $D_2$, and $D_3$ terms in (8). From (9) we have the dispersion equation

$$a_1 + \frac{3 \omega_D}{4 \omega} a_0 - 2\sqrt{\pi} k_\perp \rho_i = -i \left[A \left(\frac{3 \omega_D}{4 \omega} a_0 \left(s - \frac{1}{2}\right) - \sqrt{\pi} k_\perp \rho_i s^2 + \frac{v_i^2}{q^2 R^2 \omega^2} a_2 s^4 \frac{k_\perp^2 \rho_i^2}{4} - \frac{3}{4} A s^4 \right) \right]^{1/2}. \tag{10}$$

Strong coupling approximation is valid for modes well localized in the ballooning space $\theta$. This assumption has been confirmed for the general case of Eq. (8) with a shooting code solution. An integral equation approach for (7) also gives similar results [20]. Solution of the dispersion equation (10) in the slab limit $\omega_D = 0$ is shown in Fig. 3. As follows from Fig. 3, the mode becomes unstable when the magnetic shear parameter exceeds some threshold value that depends on $\eta_i$. The mode growth rate decreases for lower values of the magnetic shear according to (10).

The finite value of the toroidal drift frequency provides an additional destabilization mechanism, creating a toroidal branch with a larger growth rate as shown in Fig. 4. Note that there are two unstable branches. For larger values of $\omega_D$ both branches merge into an interchange type mode with a growth rate $\gamma = \left[3(1 + \eta_2/2)\omega_{Dp}/(16\sqrt{\pi} L_\parallel)\right]^{3/2}$. The toroidal branch is weakly affected by the magnetic shear so that the growth rate remains significant even in the region of the negative shear as shown in Fig. 4.

![FIG. 3. Ion short wavelength mode in the fluid slab limit, $\omega_D = 0$. Real (solid line, left panel) and imaginary (dashed line, right panel) parts of the normalized wave frequency $\omega/\omega_{k_the}$ are shown as functions of the shear parameter $s$ for $\eta_i = 3$, $\alpha_i = k_i \rho_i k_i v_i/(2 \omega_*) = 0.28$, and $k_i \rho_i = 4\sqrt{2}$. Here $k_i = 1/(qR)$.](image)
New short wavelength temperature gradient driven modes can produce a significant level of anomalous transport. A simple mixing length estimate gives for the toroidal interchange mode \( D_1 \approx \rho_e^2 v_{thi} \sqrt{\varepsilon_r \beta} / (L_n a_m^2) \), where a numerical factor \( a_m \) corresponds to the wave vector with the maximal growth rate, \( k_{\perp} \rho_i \approx a_m > 1 \). The toroidal short wavelength mode is closely related to the “ubiquitous” mode that includes the effect of trapped electrons [18]. Trapped electrons will further increase the mode growth rate and, respectively, the turbulent diffusion associated with such a mode. One of the most interesting features of the short wavelength ion mode is its ability to produce a finite level of the electron transport because the mode growth rate remains finite well into the region \( k_{\perp} \rho_i \gtrsim \beta > 1 \). The electron transport produced by the short wavelength modes will be substantially larger than that of the standard ETG modes. It is interesting to note that, for moderate plasma pressures \( \beta = 1\% - 2\% \), the ion short wavelength mode extends into the region \( k_{\perp} \approx c / \omega_{pe} \) where the electron transport may be further increased by the electromagnetic effects [9,21]. A new mode existing in the intermediate region may provide coupling between standard ITG and ETG modes thus leading to a multiverlength-scale turbulence state [22].

In summary, we have identified new short wavelength branches of the temperature gradient driven modes. These modes are closely related to acoustic type modes that are destabilized by both the temperature gradient effects and plasma toroidicity and persist in the region of the weak/ negative magnetic shear. A modified plasma response in the region of \( k_{\perp} \rho_n \gg 1 \) is essential for a new instability.

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