Results From Improved Monte Carlo Calculations of Auroral Ion Velocity Distributions

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We have computed ion velocity distributions for the auroral ionosphere with the use of a Monte Carlo computation scheme that includes a detailed description of the interaction between ions and neutrals. This means that both the speed and angular dependence of the interaction are accounted for in a self-consistent manner. We have used the model to deal with several types of interactions all at once and to describe quantitatively how the velocity distribution evolves in the presence of a mixture of neutral constituents. The model also contains an ad hoc procedure to deal with the effect of chemistry, which is important for periods during which N₂ ions are abundant. We have found that the shape of NO⁺ ion velocity distributions reaches an asymptotic limit for electric fields reaching about 125 mV/m, that the shape of O⁺ and N₂⁺ ion velocity distributions may be very difficult to model analytically for very large electric field strengths, but that conversely, analytical theories should do an adequate job for fields up to 100 or 150 mV/m for these two ion species. We have, among other things, presented our results in terms of an ion temperature anisotropy, showing how this anisotropy, or the partition parameters that are sometimes used to describe it, change with electric field strength. We have found in the process that the anisotropy of O⁺ ions is noticeably greater than what has been assumed to be the case in the past. Finally, we have found that the shape of the distribution is sensitive to the presence of a secondary neutral constituent. For example, the presence of just 25% atomic oxygen is sufficient to give the O⁺ distribution a detectably greater toroidal character.

1. INTRODUCTION

The presence of strong electric fields at high latitudes produces ion flows (E x B drifts) that can often be supersonic with respect to the neutral gas with which they collide. In the presence of strong enough electric fields the ion velocity distribution can, as a result, depart significantly from the familiar Maxwellian shape associated with gas equilibrium conditions.

The problem of theoretically determining the basic shape of an ion velocity distribution in strong nonequilibrium conditions in the auroral regions was first investigated by Cole [1971], Schunk and Walker [1972], and finally in a series of papers by St-Maurice and Schunk [1973, 1974, 1976, 1977]. It was found that for a magnetized plasma and for a constant and uniform electric field the combined sustained action of the Lorentz force and of collisions with neutrals tends to deplete the number of ions at the E x B drift point in the velocity distribution, and to produce strongly toroidal ion distributions in velocity space under extreme conditions. A review by St-Maurice and Schunk [1979] provides a more detailed explanation of the physical principles at work and of the various theoretical approaches that had been used until that time.

The initial theoretical studies on the subject were followed by the first attempts to understand how "non-Maxwellian" ion velocity distributions, as they became known, affect the interpretation of data in the high-latitude ionosphere. St-Maurice et al. [1976] studied retarding potential analyzer (RPA) curves from the Atmosphere Explorer C satellite and found that not only were non-Maxwellian features detectable under strong electric field conditions but also an erroneous Maxwellian analysis of data obtained during disturbed conditions could lead to a serious misinterpretation of the geophysical parameters that were being retrieved by the instrument. Raman et al. [1981] then extended the application of the work to incoherent scatter radars for the first time. They found that a non-Maxwellian signature on the spectra should be detectable and from a number of simulations determined that, once again, the retrieval of geophysical parameters (particularly species temperatures, and to a certain extent, the ion density) was going to be affected by the erroneous assumption of a Maxwellian ion velocity distribution in strong nonequilibrium conditions.

The early discovery of non-Maxwellian signatures in the satellite data and the finding that incoherent scatter data were likely to also be affected led to a renewed interest in the theoretical description of the distribution function. By the late 1970s a closed form solution had been obtained using a simple relaxation model for collisions, and the physics of what that model yielded was understood. Analytical expansion schemes had also been devised, and it was recognized that an orthogonal polynomial scheme with a clever choice of a weight function would yield satisfactory results provided the distortions from the weight function were not too large. It had been recognized that a bi-Maxwellian was a good zeroth-order choice for the distribution function (or weight function) and that the resulting expansion in terms of velocity moments would yield satisfactory results provided the electric field was "small enough". An expansion in terms of a toroidal zeroth-order velocity distribution had also been devised, but had not been carried through for a realistic case. On the other hand, the satellite data had indicated that
the one-dimensional ion velocity distribution obtained by integrating in a direction perpendicular to the geomagnetic field was similar to what a simple relaxation collision model produced. The measured O⁺ distributions had, in fact, a tendency to be doubly humped. However, the data had the important distinction from the simple theory that the toroidal features at a given electric field strength were much less pronounced than the simple model predicted, with O⁺ ions being more doubly humped than NO⁺ ions. From then on, a parameter called $D^*$ (which was really the Mach number of the ions until then) became $D^*$. The latter parameter was used as a shape parameter to describe the toroidal character of the distribution in terms of the Bessel function of order 0 and of an exponential toroidal function.

The search for improved theoretical schemes was carried on mostly by Hubert in a series of papers in the early 1980s [Hubert, 1982a, 1982b, 1982c, 1983, 1984a, 1984b]. He studied how to optimize the choice of a particular zeroth-order distribution so as to accelerate the convergence of a series expansion. In particular, he pointed out that for moderate electric fields the zeroth-order bi-Maxwellian could produce speedier convergence if its two temperatures were not chosen to be the actual physical temperatures but were rather chosen so as to reduce the contribution from the terms that were dropped in the orthogonal polynomial series expansion. He also formally devised a mathematical scheme that could handle larger electric fields by using a zeroth-order distribution function that was toroidal. His scheme produced zeroth-order distribution functions that were indeed very close to what had been obtained with the EPA measurements reported by St-Maurice et al. [1976].

While Hubert was able to improve greatly the convergence of the polynomial method with his toroidal expansion, he still had to face some inherent limitations with this method. One of these was that the collision frequency was assumed to be independent of velocity (the so-called "Maxwell molecule" model). While this model is actually good for a first-order description of ion-molecule interactions, St-Maurice and Schunk [1977] had pointed out that there was a significant non-Maxwell molecule contribution in the interaction potential because of the energies involved in ionospheric situations. This meant that the ratio of the parallel to perpendicular ion temperatures was not easy to determine and that the expansion schemes used had some basic inaccuracies that could only be remedied by expanding again, this time in terms of the speed-dependent collision frequency. A second limitation which required investigation was that the zeroth-order description of the distribution function was always chosen to be Maxwellian in the direction of the magnetic field, even though the solutions indicated that most of the time the one-dimensional ion velocity distributions along the magnetic field were more peaked and had a faster tail than the pure Maxwellian form that the model had in this direction.

The kind of uncertainty borne by analytical theory led to the development of a third approach to the study of auroral ion velocity distributions, namely, to the first Monte Carlo calculations of the distribution function by Barakat et al. [1983]. These calculations have the advantage of not being limited to particular electric field strengths and of providing an accurate description of the distribution function equally well in all directions. This is because the Monte Carlo method is such that it accurately monitors the ions in the region of velocity space where they spend most of their time. The limitations of these calculations are that they are time consuming (limiting their usefulness for routine applications involving measurements), are limited in accuracy to a couple of thermal speeds around the mean drift point, and depend for their accuracy on the correctness of the ion-neutral interaction potential that is being used. The last two limitations are in fact similar to the limitations faced by analytical theories, but with the important point that they are far easier to remedy with a Monte Carlo method than with the analytical method. With the relatively simple hard sphere type of collision model that was used at the time, Barakat et al. [1983] nevertheless were able to show that the distribution function of O⁺ ions was indeed toroidal and continued to become increasingly toroidal as the electric field strength increased. No quantitative comparisons with theory or measurements were attempted at the time.

With the advent of the EISCAT (European Incoherent Scatter) radar system, the second part of the 1980s led to a third wave of studies, this time strongly observationally driven. With its tri-static receivers the EISCAT radar is ideally suited to detect anisotropies in the ion temperature, and indeed it did provide the initial evidence for such anisotropies early on [Perraut et al., 1984a, 1984b; Flé et al., 1986; Løvhaug and Flé, 1986]. As the radar technique improved and the number of chances to see very large electric field signatures increased, the non-Maxwellian signatures per se were also finally convincingly detected for the first time, particularly in spectra for which very large elevation angles had been used to study the cleft region [e.g., Winser et al., 1986; Lockwood et al., 1987; Moorcroft and Schlegel, 1988]. The signatures were very similar to what had been predicted theoretically first by Raman et al. [1981] and later on by Hubert et al. [1984b]. These experimental results indicated once and for all that non-Maxwellian signatures had to be dealt with in radar studies of large electric field events if a proper interpretation of the geophysical parameters was to be retrieved for these events. A number of quantitative studies of the radar measurements followed. Many of them concentrated on getting a proper description of the shape of the ion velocity distribution as part of the analysis, using the $D^*$ parameter mentioned earlier [Svanto et al., 1989a, b; Lockwood et al., 1989]. The results were in significant agreement with theoretical predictions. Other studies concentrated on the observed temperature anisotropies and on the ion temperature variations as a function of electric field under more moderate electric field conditions [e.g., Glattor and Hernandes, 1990; I. W. McCrea et al., Derivation of ion temperature partition coefficients from the study of ion frictional heating events, submitted to Journal of Geophysical Research, 1991, hereafter cited as I. W. McCrea et al., submitted, 1991]. Once again, very reasonable agreement with theory seemed to be observed.

On the theoretical front, the need was felt to orient the calculations to the computation of incoherent scatter radar spectra, given the fact that the large number of parameters that are being retrieved from the spectra leads to ambivalent results at times (a spectral shape is strongly influenced by the ion composition, the electron to ion temperature ratio, and, finally, the toroidal features of the ion velocity distribution). Thus, using Hubert's analytical method, Hubert and Lathuilière [1989] and Lathuilière and Hubert [1989] did a careful study of the effect of composition on the spectra.
Around the same time, Suuranto [1988, 1990] was optimizing the retrieval techniques to recover $D^*$ from the spectrum. Kikuchi et al. [1989] were using the Monte Carlo results from the program developed by Barakat et al. [1983] to study how the spectral shape would evolve as a function of aspect angle and electric field. Kikuchi et al. proposed to use a toroidal description along the line of sight all the way up to directions parallel to the magnetic field, and they suggested that for moderate fields at least, the characterization of the doubly humped features by the superposition of two Maxwellsians along the line of sight appeared to be adequate; this meant that a considerable gain in analysis speed could be had for a first-order study of temperatures under disturbed conditions.

In the very recent past the field has evolved further: the effect of ion-ion collisions on the partition parameter (or the temperature anisotropy) is now being considered in some detail both theoretically and experimentally [Tereshchenko et al., 1991; Kinzelin and Hubert, 1990; I. W. McCrea et al., submitted, 1991]. Incoherent radar computations for the altitude region around 120 km, where the ion velocity distribution is still non-Maxwellian, but also three-dimensionally anisotropic, have been performed by Cooper and Kohl [1990a,b]. Higher up, the passage to a medium free of collisions with neutrals under strong electric field conditions is currently under study [St-Maurice et al., 1989], and the effect of large electric field shears on the distribution is also being considered [Winkler and St-Maurice, 1988, and manuscript in preparation, 1992]. On the purely analytical front, Shizgal and Hubert [1989] have been going back to a spherical polynomial expansion ("Burnett functions") centered on the neutral frame rather than a cylindrical one based on the ion frame of reference. Finally, Hubert and Barakat [1990] and Barakat and Hubert [1990] have studied the convergence of the analytical method by comparing its results with Monte Carlo results, using the velocity moments provided by the Monte Carlo calculations themselves. In that comparison they found the analytical method to be very accurate for one-dimensional distributions, even with the limited number of polynomials that can be used in practice. In the case of the three-dimensional distribution the fit gave only the general shape.

In this paper we have refined the Monte Carlo method on several fronts. To start with, we have been using a more accurate collision model to describe the ordinary interactions between ions and neutrals. Second, we have for the first time used the Monte Carlo method to study how the shape of the distribution is affected by a mixture of neutral constituents. Third, we have added the effect of chemistry on the short-lived $N_2^+$ ions in order to see if we could account for the observation by Farmer et al. [1988] that the molecular ions in the middle of the F region could be more non-Maxwellian than $O^+$ ions under special conditions for which $N_2^+$ would dominate. We have also pushed the study to fields as large as 300 mV/m to see, among other things, if the shape of the NO$^+$ velocity distribution really stops evolving when the electric field increases. We have also studied how the partition parameter changes with electric field and how the mean ion temperature deviates from the Maxwell molecule balance that is normally used in ionospheric studies. Finally, we have followed up on the Kikuchi et al. [1989] suggestion to use a toroidal or a doubly humped description for all directions by fitting a variety of one-dimensional distribution functions to the two models that these authors suggested. We have, in the process, assessed when such models should be used and when they should not.

Section 2 presents the collision models used and explains how a Monte Carlo technique has been used in the current work to generate ion velocity distributions. In section 3 we present some of the three-dimensional ion velocity distributions produced by these calculations. We then present, in section 4, one-dimensional ion velocity distributions taken along various lines of sight and compare these to the one-dimensional distributions generated by two fitting techniques used in earlier work in order to determine optimum $D^*$ values as a function of electric field strength and aspect angle. Section 5 contains a summary of the results.

2. Collision Models and Monte Carlo

Technique

The Monte Carlo technique is used to approximate the solution of a physical problem by using random sampling. We implemented it here to produce a simulation of auroral ion velocity distributions by recording the velocity of a representative (test) ion as it undergoes a large number of collisions with neutrals. Since the simulations were for a system which is in a quasi-steady state, we used the ergodic hypothesis, namely, we assumed that time averages can be equated to ensemble averages. The present Monte Carlo study is in fact an extension of the Monte Carlo study by Barakat et al. [1983]. The basic approach is the same as was taken in that earlier work, except for the fact that we have used a more comprehensive collision model to describe the interaction between ions and neutrals so as to keep pace with the progress, both theoretical and observational, that has been made in recent years.

In the following subsections we give a more detailed overview of the simulations. Briefly stated, a starting point for the simulation was to determine the time interval between successive collisions using a properly weighted random number generator [Lin and Bardsley, 1977]. Note that between collisions the test ion velocity was tracing out circles in velocity space. These circles are centered on the $E \times B$ drift point, $V_d$. When a collision occurred, the ion velocity after the collision was determined by using another set of random numbers having statistical properties dictated by the choice of the collision model (much more on this below). We then used the ion velocities to record the time spent at a given velocity in a two-dimensional grid with velocity coordinates parallel and perpendicular to the geomagnetic field and centered at $V_d$. We specialised our calculations to F region situations; this means that the azimuthal angular dependence in velocity space was neglected, since the ion collision frequency is then much smaller than the ion cyclotron frequency [e.g., St-Maurice and Schunk, 1979]. Finally, the ion velocity distribution was calculated by assuming that the time spent by the ion in each bin of the velocity grid, divided by that bin's volume, was proportional to the ion velocity distribution at the center of the bin [e.g., Barakat and Schunk, 1982]. We now give more details about some of the new features that were used in our simulations.

2.1. The "4-12" Collision Model

When any ion collides with a neutral, the scattering process can be satisfactorily modeled with the use of a long-
range polarization attraction (potential $\propto 1/r^4$) and a short-range repulsion [e.g., Mason, 1970]. For our study, and following the work by Mason and Schamp [1958], we have chosen the potential energy of interaction $V(r)$ between an ion and a neutral at a separation distance $r$ to be given by even power laws, so as to facilitate the collision probability calculations. We therefore chose the potential of interaction to be given by the expression

$$V(r) = \frac{\epsilon}{2} \left[ (1 + \gamma) \left( \frac{r_m}{r} \right)^{12} - 4\gamma \left( \frac{r_m}{r} \right)^6 - 3 (1 - \gamma) \left( \frac{r_m}{r} \right)^4 \right] \quad (1)$$

where $r_m$ is the value of $r$ for which $V(r)$ is a minimum, $\epsilon$ is the depth of the potential well, and $\gamma$ is a measure of the relative strengths of the different attractive forces. The inverse fourth-power term accounts for the attraction between an ion and the dipole it induces in the polarizable neutral molecule. The inverse sixth-power is a combination term of contributions from both the charge-induced quadrupole and the London dispersion energy. We have chosen $\gamma = 0$ and therefore neglected the sixth-power effects. This is partly to limit the number of parameters presented here to a manageable number and partly because at least for earlier mobility measurements in the gases that we have considered, the retrieved information has been too limited to tell whether the $1/r^6$ potential is important or not [Mason, 1970]. As for the choice of the power 12 in the repulsion potential, we draw again on the discussion by Mason [1970], who found this model to be about right in the light of existing transport measurements. In other words, it was properly softer than a rigid sphere ($1/r^n, n \to \infty$) and harder than, say, a $1/r^8$ law, which mobility measurements indicated was too soft a model. Both $r_m$ and $\epsilon$ are specific to the colliding partners, with $r_m$ of the order of $10^{-8}$ cm and $\epsilon$ of the order of 0.1 eV (the actual "standard" values that we chose for the gases of interest here are given in Table 1). In the rest of the text we refer to equation (1) with $\gamma = 0$ as the "4-12" interaction.

The use of a physical model for the potential of interaction between a neutral and an ion allows us to properly calculate a scattering angle for a given impact parameter. The effects due to the velocity dependence of the scattering cross section can therefore be described consistently, that is, in physical terms that require no heuristic model, except of course for the model for the potential of interaction itself. In our simulations we also assumed that the energy of interaction was large enough that we could compute classically the scattering angle $\chi$ in the center-of-mass frame for the interaction between the test ion and a neutral. In other words we used the well-known relation

$$\chi(b, g) = \pi - 2b \int_0^\infty \left[ 1 - \frac{V(r)}{\mu g^2/2} \frac{b^2}{r^2} \right]^{-1/2} \frac{dr}{r^2} \quad (2)$$

where $b$ is the impact parameter, $\mu$ is the reduced mass, $g$ is the initial relative speed of the colliding particles, and $r_c$ is the distance of closest approach that can be determined from the relative energy.

2.2. Resonant Charge Exchange Collisions

When an ion collides with its parent neutral gas, there is, in addition to the ordinary long-range polarization and short-range repulsive forces, a definite possibility that an electron will switch from the neutral to the ion, thus causing an effective change of identity. This process by itself corresponds to backscattering ($\chi = \pi$) in the center-of-mass system. This process is termed "resonant charge exchange" or RCE. The RCE interaction typically has a range that is shorter than the polarization interaction at low energies but is larger than the short-range repulsion forces at high energies. As a result it tends to dominate at some point as the relative energy, or $g$, increases [e.g., Banks and Kockarts, 1973].

One difficulty with modeling an RCE interaction with the Monte Carlo approach is that, by symmetry, the charge exchange process cannot occur for every collision of an ion with its parent neutral. This is because whenever an ion and its parent neutral get close enough for an RCE collision to be possible (at some impact parameter $b < b_c$, where $b_c$ is a critical impact parameter beyond which the RCE probability decreases very rapidly [e.g., Stubbe, 1968]), the charge exchange process has to be unsuccessful for half of the time. This is because once involved in this chemical type of situation, the electron cannot determine which of the nuclei it came from. As a result we have modeled the probability of an RCE occurrence with the function

$$P_{ee} = \frac{1}{2} \quad \text{for} \quad b < b_c (g)$$

$$P_{ee} = 0 \quad \text{for} \quad b > b_c (g)$$

Note that since the critical impact parameter $b_c$ was assumed to be roughly a boundary between no RCE and all RCE, we have assumed that it was related to the total cross section via the relation $b_c^2 = Q_{ee}^2$ where $Q_{ee}^2$ is the total cross section for charge exchange processes and is discussed below.

2.3. A "Collision" Model for Short-Lived Chemical Species

While the ions NO$^+$, O$^+$, and O$_2^+$ all undergo a large number of collisions with neutrals before they chemically react and disappear, individual N$_2^+$ ions can be very reactive and at times only survive for a few collisions with neutrals before reacting with other constituents of the atmosphere and disappearing (using, for example, the reaction rates given by Banks and Kockarts [1973, chapter 10],

<table>
<thead>
<tr>
<th>Colliding Particles</th>
<th>$\epsilon$, eV</th>
<th>$r_m^*$, $\times 10^4$ cm</th>
<th>$A$, $\times 10^4$ cm</th>
<th>$B$, $\times 10^4$ cm</th>
<th>$T$, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO$^+$ - O</td>
<td>0.110</td>
<td>2.44</td>
<td>0.00</td>
<td>0.00</td>
<td>1000</td>
</tr>
<tr>
<td>N$_2^+$ - N$_2$</td>
<td>0.110</td>
<td>2.93</td>
<td>10.37</td>
<td>0.64</td>
<td>1000</td>
</tr>
<tr>
<td>O$^+$ - N$_2$</td>
<td>0.110</td>
<td>2.93</td>
<td>0.00</td>
<td>0.00</td>
<td>1000</td>
</tr>
<tr>
<td>O$^+$ - O</td>
<td>0.075</td>
<td>2.75</td>
<td>10.99</td>
<td>0.95</td>
<td>1000</td>
</tr>
</tbody>
</table>
we find that the chemical lifetime of \( \text{N}_2^+ \) ions can roughly be anywhere between 5 and 20 times longer than an elastic collision time, depending on conditions. Although such "collisions" are inelastic, we can still roughly examine the effects of a relatively short chemical lifetime by using a relaxation collision model. This idea, which originally came from P.B. Hays (private communication, 1975), is based on the fact that in the quasi-steady state situation, on average, a new \( \text{N}_2^+ \) ion is created whenever another \( \text{N}_2^+ \) ion disappears. These ions are produced by soft electron impact. These impinging electrons do not affect the kinetic energy of the \( \text{N}_2 \) by much, so that \( \text{N}_2^+ \) ions are formed with a velocity distribution which mimics very well that of the neutrals. However, these newly created ions do not have a near zero velocity for long since they immediately become subject to the action of the Lorentz force. Furthermore, in the region where they are created, we have \( \nu_{\text{in}} < \Omega_i \), which means that they perform many orbits in velocity space before a collision, and therefore a chemical reaction, will occur. Consequently, the short-lived individual \( \text{N}_2^+ \) ions behave as if they were not disappearing via a chemical reaction but rather as if they were suffering charge exchange collisions at a rate equal to their chemical lifetime. This allows the chemical process to be modeled rather accurately by the simple relaxation model that was used in the work of St-Maurice and Schunk in the series of papers mentioned in the introduction. According to that model, the velocity distribution of ions immediately after collisions with neutrals is the velocity distribution of these neutrals (the model also assumes that the collision frequency is constant).

### 2.4. Total Cross Sections

It is essential in our work to obtain a value for the total collision cross section in order to determine if we have a collision or not, that is, in obtaining the time spent by an ion between collisions. In the context of our simulations the total cross section is given by

\[
Q_T = \pi b_{\text{max}}^2
\]

(4)

where \( b_{\text{max}} \) is the maximum impact parameter for which a "significant" deflection of the ion will occur. We use the word "significant" because, in principle at least, the total collision cross section for a "4-12" interaction could be viewed as infinite since the potential drops off as \( r \) increases but does not vanish unless \( r \to \infty \). This problem is remedied by choosing a cutoff impact parameter \( b_{\text{max}} \) beyond which the 4-12 interaction is considered insignificant. This cutoff is chosen based on the amount of scattering required to change the ion velocity enough to move it into a new velocity bin.

For the RCE interaction the total cross section is given by

\[
Q_T^R(E) = (A - B \log_{10} E)^2
\]

(5)

where \( E \) is the kinetic energy of relative motion in electron volts [Banks and Kockarts, 1973, p. 221]. Both \( A \) and \( B \) are constants which depend on the species of the interacting particles and were determined from beam experiments [e.g., Banks, 1966]. It is important to recall, however, that (5) was obtained under the assumption that a collision always results in a transfer of an electron. And yet, as we already mentioned, an electron exchange can only happen with a probability of 50%, so that, in order to compare the total resonant charge exchange cross section with that of any of the other collision models, \( Q_T^R \) must first be doubled. Once this is taken into account, we can determine a maximum impact parameter for an RCE collision by equating equations (4) and \( 2 \times (5) \).  

### 2.5. Collision Frequencies and the "Null Collision" Approach

For a collision with a collision frequency \( \nu \) that is independent of the relative velocity, the probability that a test particle will suffer no collision for a time interval \( t \) is given by

\[
P(t) = \exp(-\nu t)
\]

(6)

If the collision frequency were constant, all we would need to do to determine the time interval between collisions would be to generate that interval using

\[
t = -\frac{1}{\nu} \log(R)
\]

(7)

where \( R \) is a random number with a uniform probability between 0 and 1 [Lin and Bardsley, 1977].

However, for the collisions that we are considering, the total cross section of interaction is speed-dependent, and the collision frequency is consequently a function of energy. This is because the collision frequency is given by

\[
\nu_{\text{in}} = n_n g Q_T(g)
\]

(8)

where \( n_n \) is the neutral density, \( g \) is the relative velocity between the ion and the neutral, and \( Q_T \) is the total cross section that has already been discussed in the previous subsection. The time interval therefore depends on the continually changing relative velocity of the colliding particles, so that the simple expression for the probability that the time between two collisions has the value \( t \) given by (7) is no longer valid.

To correct the speed dependence difficulty, we first determine a maximum collision rate, \( k_{\text{max}} \). We then apply the constant collision frequency solution to this problem. This is done by dividing the determination of time between collisions into two steps. We begin by defining a collision occurrence rate as

\[
k(g) = g Q_T(g)
\]

(9)

We then use the maximum collision rate \( k_{\text{max}} = [g Q_T]_{\text{max}} \) along with equations (7) and (8) to obtain the minimum time between collisions.

In a second step we must correct for the fact that the actual collision rate for a given \( g \) is less than \( k_{\text{max}} \); this leads to collision time intervals that are longer than the time given by equation (7). In order to obtain the desired correction, we first consider the probability, \( P_r(g) \), that the "collision" be real. This probability is given by

\[
P_r(g) = \frac{Q_T(g)}{Q_T^{\text{tot}}(g)} = \frac{g Q_T(g)}{k_{\text{max}}}
\]

(10)

where \( Q_T(g) \) is the cross section for an actual ("real") collision to occur and

\[
Q_T^{\text{tot}}(g) = \frac{k_{\text{max}}}{g}
\]

(11)

For cross sections greater than \( Q_T(g) \) and less than \( Q_T^{\text{tot}} \), no collision actually occurs, but the time interval is recorded.
We then simply sum all the times generated by any number of these "null" collisions until the criterion for a "real" (actual) collision is met. This sum of times is the time between two real collisions and compensates for the actual speed dependence of the collision frequency.

2.6. The Handling of "Mixed" Collision Processes

When an ion collides with its parent neutral, it undergoes more than just one type of interaction, since the interaction process is described by a mixture of 4-12 interactions, RCE interactions and, in the case of N2, even chemical reactions, all occurring simultaneously. We determine which process or processes actually occur by comparing the maximum impact parameter for each of the processes to the impact parameter of the incoming ion. This is done by assuming that whenever \( b < b_{max}(\text{of a process}) \), that particular process actually takes place. As a result, recalling that \( Q_T = n^2 \) and using equation (10), we can rewrite the probability for a real collision to occur as

\[
P_r(\gamma) = \frac{\max \left[ b_{max}(\text{RCE}), b_{max}(4-12), b_{max}(\text{chem}) \right]}{b_{tot}^2} \tag{12}
\]

where \( b_{tot}^2 = Q_T^{tot}/\pi \). We then choose the impact parameter for our collision by using

\[
b^2 = Rb_{tot}^2 \tag{13}
\]

where \( R \) is a random number with a uniform probability between 0 and 1. If \( R < P_r(\gamma) \), then an actual collision occurs, and we proceed to determine which type(s). Otherwise we have a "null" collision, record the time interval, and move on to the next collision.

2.7. The Question of Overall Accuracy in the Calculations

One final question to address is the accuracy of the calculations. Putting aside questions about the validity of the collision models used, and about other assumptions inherent to the physics being modeled here, we still have to ask what uncertainties are related to the method of computation itself. For example, there is always a danger that the random number generator used could have cycles, in which case the accuracy of the calculations does not improve if the calculation uses a number of calls to the random number generator that exceeds the length of the cycle; to reduce this possibility we periodically reseed the random number generator using a procedure based on clock time and not on numbers from the generator.

Another problem is with the statistics themselves. A compromise had to be struck between the desired accuracy level and the length of time used for the computation. Our objective was to choose the standard number of "real" collisions in a run so as to get an accuracy better than 10% at the two thermal speeds level. The accuracy achieved was determined by monitoring the changes in a velocity bin as a function of the total number of collisions. We found that with \( 2 \times 10^6 \) collisions we could reach the desired level of accuracy at two thermal speeds. Also note that for highly toroidal features the accuracy had to be defined not in terms of thermal speeds but in terms of how small the distribution function in a particular bin was, relative to the bin where the peak in the velocity distribution was reached. The "two thermal speeds or greater" region translated to regions where the distribution function was 1% or less of the peak value.

To test the validity of our accuracy determination we used a collisional cross section in the Monte Carlo computation that was such that it would reproduce the pure relaxation model solution of St-Maurice and Schunk [1974]. This cross section, the origin of which is described by St-Maurice and Schunk [1977], consists of a backscatter collision process in the center-of-mass system, with a Maxwell molecule speed dependence. As a specific example, we ran a test for which the electric field was 300 mV/m and the neutral temperature 1000 K. We compared the Monte Carlo distribution with the analytical description. The differences between the numerical distributions and analytical ones were actually less than 5% in the boundary region for which the distribution function was 1% of the maximum. The differences were less than 1% in regions for which the distribution was of the order of 10 times greater.

3. Ion Velocity Distributions

In this section we present ion velocity distributions produced with the method discussed in section 2. The results are for various ions and for various electric field strengths and neutral background compositions, with particular emphasis on situations that have not been considered in the past. Along those lines we have also studied the effect of a chemical lifetime of varying length on the velocity distribution of N2 ions. Each of the simulations presented here consists of \( 2 \times 10^6 \) "real" collisions. Furthermore, unless otherwise stated, the neutral temperature, geomagnetic field strength, and background neutral density have been held constant at 1000 K, 0.5 gauss, and \( 10^{10} \) particles cm\(^{-3} \), respectively.

3.1. Basic Results: Logarithmic Contour Representation

A most natural way to display our results and become familiar with their basic meaning is to display them in the form of a logarithmic contour plot similar to what was used earlier by Barakat et al. [1983]. In such displays we plot the natural logarithm of the ratio of the number of ions in a given velocity bin to the maximum value of the distribution as a function of \( v_{\perp} \) and \( v_{\parallel} \) (where \( v_{\parallel} \) is the component of \( v \) along the geomagnetic field). We refer to these as "log" plots. Notice that the origin of the \( v_{\perp} \) axis is at the \( E \times B \) drift point in these cases. Also, when considering the simulations, it is important to recall that the results are only significant for speeds less than about two ion thermal speeds. As the speed increases much beyond this point, the statistics become poor since there are then too few collisions per velocity bin.

We first examined the effect of a changing electric field on the ion velocity distribution. We start our presentation with NO\(^+\) colliding with O, which is relatively simple conceptually speaking, since only one type of interaction, the "4-12", occurs in that case. In our "log plot" format, Figure 1 shows the effect of increasing the electric field from 50 mV/m to 300 mV/m for NO\(^+\) ions. In agreement with previous theoretical work and with observations, Figure 1 does show that as the electric field increases, the ion velocity distribution flattens out in the center along the \( v_{\perp} \) axis. In addition, as was also suggested by earlier theoretical work and observations [St-Maurice et al., 1976; Hubert, 1983], we find that although the ion temperature (not shown here) continues to increase beyond 150 mV/m, no noticeable additional evolution can be detected in the shape of the NO\(^+\) velocity
distribution. The shape appears to "saturate" in this case. A brief discussion of the reasons for this saturation is found in the appendix.

The next situation that we examined was that of an ion colliding with its parent neutral. In this conceptually more complicated case, both 4-12 and RCE processes occur simultaneously. To help see the effect of such a mixed model on the velocity distribution we first present, in Figure 2, log plots of the ion velocity distributions for (Figure 2a) pure 4-12 and (Figure 2b) pure RCE interactions for an electric field of 100 mV/m. Unlike the 4-12 case, the ion velocity distribution for the pure RCE case exhibits highly toroidal features, as expected from earlier work on the subject [e.g., St-Maurice and Schunk, 1979]. Also as expected, the result of combining the two collisional processes for a given strong electric field is to create distributions that are more toroidal than for the pure 4-12 case but less toroidal than for the pure RCE case. This is shown in Figure 3a for the 100-mV/m case of Figure 2. Figure 3b also shows the ion velocity distribution for the same mixed model, but for an electric field of 250 mV/m. This distribution appears much more toroidal than for the case where $E = 100$ mV/m. This
means that the RCE features are responsible for a continuous evolution of the distribution function with electric field strength, unlike the pure 4-12 case shown earlier, which saturated around 150 mV/m. Barakat et al. [1983] had also found that the shape of a mixed RCE model continued to evolve with electric field strength, even though they were using a simpler collision model in order to take account of the 4-12 interaction.

In actuality, the background neutral gas in the ionosphere is composed of several different species. We therefore let the ions collide with more than one type of neutral and studied the effect of a varying neutral composition on the ion velocity distribution. Figure 4 shows a comparison between (Figure 4a) O⁺ colliding with O using our mixed collision model and (Figure 4b) O⁺ colliding with a background composed of 75% O and 25% N₂. As should again have been expected on qualitative grounds, adding molecular nitrogen to the background in this case erodes the strong toroidal features associated with the RCE dominance in collisions with the parent gas, namely, atomic oxygen in this particular study.

The trend shown in Figure 4 continues as we decrease the amount of O in the background. Figure 5 shows (Figure 5a) O⁺ colliding with N₂ and (Figure 5b) O⁺ colliding with a background of 25% O and 75% N₂. With the lower background atomic oxygen density, the mixed background case resembles more that of O⁺ colliding with a pure N₂ background than O⁺ colliding with O. On the other hand, it is also interesting to note that we can already start to see the effects of the O⁺—O RCE interactions on the distribution function even when only 25% of the neutral gas is made of atomic oxygen.

In the case of N⁺ we allowed for one additional final level of complexity. Namely, as explained in section 2, we allowed N⁺ to rapidly undergo a chemical reaction with other constituents in the ionosphere, thus creating the disappear-
Fig. 4. Contours of $\ln(f_i/f_{imax})$ for (a) O$^+$-O collisions and (b) O$^+$ colliding with 75% O and 25% N$_2$. Both figures use $E_L = 100$ mV/m and a mixed collision model.

Fig. 5. Contours of $\ln(f_i/f_{imax})$ for (a) O$^+$-N$_2$ collisions and (b) O$^+$ colliding with 25% O and 75% N$_2$. Both figures use $E_L = 100$ mV/m and a mixed collision model.

3.2. Ion Temperature Studies

As we have presented them, our results can so far only be used to make qualitative inferences about how various situations affect the toroidal character of the non-Maxwellian ion velocity distribution in the high-latitude F region. However, measurements require a much more quantitative study than what we have provided so far in this presentation. To start with, radars and satellite instruments are most often used to infer the ion temperature along a given line of sight. In that sense, the change in the ion temperature along a given line of sight, on average, before suffering a chemical reaction. From Figure 6 we observe that the shorter the chemical lifetime, the more the distribution exhibits the pure (as opposed to mixed) RCE toroidal features. The figure indicates that chemical reactions have an effect for chemical lifetimes of the order of 10 elastic collision times or less, which is an entirely feasible situation (the exact value of the ratio depends, in a particular geophysical situation, on the ambient electron density).
since the distribution function is anisotropic, however, the "temperature" must also change with the direction of the line of sight.

In quantitative studies of the ion temperature anisotropy in the past, the ion temperature along the magnetic field has been defined as $T_{\parallel}$ and the temperature in a direction perpendicular to the geomagnetic field as $T_{\perp}$. These temperatures have been written in terms of the electric field strength and the neutral temperature $T_n$ with the use of two partition parameters, $\beta_{||}$ and $\beta_{\perp}$. These were defined using the following equations [St-Maurice and Schunk, 1977]:

$$T_{\parallel} = T_n \left[ 1 + \beta_{||} D^2 \right]$$

$$T_{\perp} = T_n \left[ 1 + \beta_{\perp} D^2 \right]$$

where $D'$ is the ion Mach number, $(Ec/B)/\sqrt{2\kappa_b T_n m_i}$. The two partition parameters are pure numbers which depend, in principle, on the collision model used, on the ion-neutral mass ratio, and on the electric field strength. They obviously determine the extent to which $T_{\parallel}$ and $T_{\perp}$ differ. Note that when dealing with a mixture of neutral constituents the neutral mass in the Mach number calculation must formally be replaced by an average, $< m_n >$, given by [e.g., Schunk and Walker, 1972]

$$< m_n > = \frac{\sum m_i \nu_{is}}{\sum m_i + \sum \nu_{is}}$$

where the sum is over all neutral species present in the gas. One then has to use tabulated momentum transfer collision frequencies in order to compute (16).

For pure polarization interactions (pure $1/r^4$ potential or so-called "Maxwell molecule" interactions) the partition parameters do not change with energy. Furthermore, for the idealized situation modeled here, the ions are in a steady state, all gradients are neglected, and any form of interaction with other charged particles is also not considered (see St-Maurice and Hanson [1982] for an analysis of the validity of such approximations). In that case there has to be a balance between frictional heating and heat exchange with the neutrals in the ion energy balance. This in turn implies that the partition parameters have to obey the balance

$$\beta_{||} + 2\beta_{\perp} = 2$$

This balance, once used in conjunction with (14), is just another way to determine the relation between the average ion temperature $(T_{\parallel} + 2T_{\perp})/3$ and the electric field. For Maxwell molecules the relation between $T_i$ and the electric field strength is therefore purely a quadratic one. This is not the case for other forms of ion-neutral interactions [e.g., Schunk and St-Maurice, 1981; St-Maurice and Hanson, 1982].

In the vast majority of theoretical work done to date, a pure polarization collision model has been used to describe the speed dependence of the ion-neutral collisional cross section. This assumption greatly simplifies the already complicated calculations, while the speed dependence of the actual cross section is not considerably different from that of a Maxwell molecule [e.g., St-Maurice and Schunk, 1977]. Nevertheless, the partition parameters are in fact functions of the electric field strength and of the complicated ways by which various forces interact during an ion-neutral collision. They are, furthermore, functions of the ion velocity distribution itself. With a complete description of the ion velocity distribution, including its temperature anisotropy, a Monte Carlo calculation such as ours is therefore one of the easiest ways by which to study how the ion temperature and its anisotropy, and therefore the partition parameters, all vary with specific conditions. Our calculations can then be used to provide a better estimate of the mean ion temperature and its anisotropy and to compare this improved estimate
with what theoretical models based on Maxwell molecule or, say, hard sphere interactions predict. This comparison should be useful for careful ion temperature studies of the kind that are currently being done in particular with the EISCAT radar [e.g., Glatthor and Hernandez, 1990; I. W. McCrea et al., submitted, 1991].

**Results from the mean ion temperature computations.**

Table 2 shows parallel and perpendicular ion temperatures as well as the \( \beta \) values that were calculated from these temperatures for several Monte Carlo runs that used different models and combinations of ion-neutral interactions and various electric field strengths. The various runs are discussed in more detail in the rest of this paper. The sum \( \beta_{||} + 2\beta_{\perp} \) is also shown in Table 2 in order to allow for an easy comparison with the widely used Maxwell molecule results (note that from benchmark tests of the Monte Carlo calculations involving pure Maxwell molecule interactions we have found that the accuracy of the \( \beta_{||} + 2\beta_{\perp} \) sum is better than 0.5%).

In particular, it is easy to show that the number in that last column, divided by 2, gives the ratio of the average ion temperature that the more realistic Monte Carlo computation actually produces to the temperature obtained with the Maxwell molecule assumption. In the case of a mixture of neutral constituents, we had to generalize the comparison procedure a bit, by taking equation (16) plus the often-used collision frequencies presented by Schunk and Nagy [1980] in order to determine the mean mass for the \( D' \) calculation in the computation of the Monte Carlo derived partition parameters. We could not use the Monte Carlo collision frequencies in (16) because in this particular comparison we wanted the Mach number to be given by the customary definitions, not by what came out of our computer runs and their energy-dependent collision frequencies.

Table 2 indicates at a glance that the Maxwell molecule approximation actually works best as far as the average ion temperature is concerned when RCE interactions (rather

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**TABLE 2a. Values of Ion Temperatures and \( \beta \) for O* Colliding With N\(_2\) Using Pure 4-12 Collisions**

| \( E_g \) mV/m | \( T_{||} \) K | \( T_{\perp} \) K | \( T_{||}/T_{\perp} \) | \( \beta_{||} \) | \( \beta_{\perp} \) | \( \beta_{||} + 2\beta_{\perp} \) |
|---|---|---|---|---|---|---|
| 50 | 1,948 | 2,222 | 0.88 | 0.56 | 0.72 | 2.00 |
| 100 | 4,811 | 5,829 | 0.83 | 0.56 | 0.71 | 1.98 |
| 150 | 9,542 | 11,825 | 0.81 | 0.56 | 0.71 | 1.98 |
| 200 | 16,115 | 20,085 | 0.80 | 0.56 | 0.70 | 1.96 |
| 250 | 24,836 | 30,892 | 0.80 | 0.56 | 0.70 | 1.97 |
| 300 | 35,531 | 44,036 | 0.81 | 0.57 | 0.70 | 1.97 |

**TABLE 2b. Values of Ion Temperatures and \( \beta \) for NO* Colliding With O Using Pure 4-12 Collisions**

| \( E_g \) mV/m | \( T_{||} \) K | \( T_{\perp} \) K | \( T_{||}/T_{\perp} \) | \( \beta_{||} \) | \( \beta_{\perp} \) | \( \beta_{||} + 2\beta_{\perp} \) |
|---|---|---|---|---|---|---|
| 50 | 1,478 | 1,728 | 0.86 | 0.51 | 0.75 | 2.02 |
| 100 | 2,885 | 3,848 | 0.75 | 0.49 | 0.73 | 1.96 |
| 150 | 5,173 | 7,147 | 0.72 | 0.48 | 0.70 | 1.88 |
| 200 | 8,393 | 11,518 | 0.73 | 0.48 | 0.68 | 1.84 |
| 250 | 12,512 | 17,161 | 0.73 | 0.48 | 0.67 | 1.81 |
| 300 | 17,576 | 24,175 | 0.73 | 0.47 | 0.66 | 1.79 |

**TABLE 2c. Values of Ion Temperatures and \( \beta \) for NO* Colliding With O Using Pure 4-12 Collisions and \( E_g = 250 \) mV/m**

| \( T_0 \) K | \( T_{||} \) K | \( T_{\perp} \) K | \( T_{||}/T_{\perp} \) | \( \beta_{||} \) | \( \beta_{\perp} \) | \( \beta_{||} + 2\beta_{\perp} \) |
|---|---|---|---|---|---|---|
| 750 | 12,315 | 16,967 | 0.73 | 0.48 | 0.67 | 1.82 |
| 1000 | 12,499 | 17,132 | 0.73 | 0.47 | 0.67 | 1.81 |
| 1250 | 12,802 | 17,416 | 0.74 | 0.48 | 0.67 | 1.81 |
| 1500 | 13,205 | 18,820 | 0.74 | 0.48 | 0.67 | 1.83 |
| 2000 | 13,536 | 18,259 | 0.74 | 0.48 | 0.67 | 1.82 |

**TABLE 2d. Values of Ion Temperatures and \( \beta \) for NO* Colliding With O Using Pure 4-12 Collisions and \( E_g = 100 \) mV/m**

| \( E_g \) eV | \( T_{||} \) K | \( T_{\perp} \) K | \( T_{||}/T_{\perp} \) | \( \beta_{||} \) | \( \beta_{\perp} \) | \( \beta_{||} + 2\beta_{\perp} \) |
|---|---|---|---|---|---|---|
| 0.01 | 2,867 | 3,657 | 0.78 | 0.48 | 0.68 | 1.85 |
| 0.05 | 2,859 | 3,717 | 0.77 | 0.48 | 0.70 | 1.88 |
| 0.11 | 2,885 | 3,849 | 0.75 | 0.49 | 0.73 | 1.96 |
| 0.20 | 2,953 | 3,988 | 0.74 | 0.50 | 0.77 | 2.04 |
| 0.30 | 2,996 | 4,061 | 0.74 | 0.51 | 0.79 | 2.05 |
TABLE 2d. Values of Ion Temperatures and $\beta$ for $O^+$ Colliding With $O$ Using the Mixed Collision Model

<table>
<thead>
<tr>
<th>$E_{\alpha}$ mV/m</th>
<th>$T_\alpha$ K</th>
<th>$T_{\alpha \nu}$ K</th>
<th>$T_{\alpha \nu} / T_{\alpha}$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_{\alpha + 2\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1,229</td>
<td>1,848</td>
<td>0.67</td>
<td>0.22</td>
<td>0.88</td>
<td>1.98</td>
</tr>
<tr>
<td>100</td>
<td>1,746</td>
<td>4,440</td>
<td>0.39</td>
<td>0.20</td>
<td>0.89</td>
<td>1.97</td>
</tr>
<tr>
<td>150</td>
<td>2,599</td>
<td>8,807</td>
<td>0.30</td>
<td>0.18</td>
<td>0.80</td>
<td>1.97</td>
</tr>
<tr>
<td>200</td>
<td>3,719</td>
<td>14,893</td>
<td>0.25</td>
<td>0.17</td>
<td>0.90</td>
<td>1.97</td>
</tr>
<tr>
<td>250</td>
<td>5,078</td>
<td>22,765</td>
<td>0.22</td>
<td>0.17</td>
<td>0.90</td>
<td>1.97</td>
</tr>
<tr>
<td>300</td>
<td>6,842</td>
<td>32,442</td>
<td>0.21</td>
<td>0.16</td>
<td>0.90</td>
<td>1.97</td>
</tr>
</tbody>
</table>

TABLE 2f. Values of Ion Temperatures and $\beta$ for $N_2^+$ Colliding With $N_2$ Using the Mixed Collision Model

<table>
<thead>
<tr>
<th>Chemical Lifetime</th>
<th>$E_{\alpha}$ mV/m</th>
<th>$T_{\alpha}$ K</th>
<th>$T_{\alpha \nu}$ K</th>
<th>$T_{\alpha \nu} / T_{\alpha}$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_{\alpha + 2\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>50</td>
<td>1,196</td>
<td>2,588</td>
<td>0.46</td>
<td>0.12</td>
<td>0.94</td>
<td>1.99</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>1,603</td>
<td>7,457</td>
<td>0.22</td>
<td>0.09</td>
<td>0.95</td>
<td>1.99</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>2,182</td>
<td>15,544</td>
<td>0.14</td>
<td>0.08</td>
<td>0.95</td>
<td>1.98</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>1,324</td>
<td>2,568</td>
<td>0.48</td>
<td>0.14</td>
<td>0.92</td>
<td>1.99</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>1,732</td>
<td>7,344</td>
<td>0.24</td>
<td>0.11</td>
<td>0.93</td>
<td>1.98</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>2,499</td>
<td>15,378</td>
<td>0.16</td>
<td>0.10</td>
<td>0.94</td>
<td>1.98</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>1,296</td>
<td>2,524</td>
<td>0.51</td>
<td>0.17</td>
<td>0.90</td>
<td>1.97</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>1,930</td>
<td>7,230</td>
<td>0.27</td>
<td>0.14</td>
<td>0.92</td>
<td>1.97</td>
</tr>
<tr>
<td>10</td>
<td>150</td>
<td>2,893</td>
<td>15,149</td>
<td>0.19</td>
<td>0.12</td>
<td>0.93</td>
<td>1.98</td>
</tr>
</tbody>
</table>

TABLE 2g. Values of Ion Temperatures and $\beta$ for $O^+$ Colliding With a Mixed Background

<table>
<thead>
<tr>
<th>Neutral Species</th>
<th>$E_{\alpha}$ mV/m</th>
<th>$T_{\alpha}$ K</th>
<th>$T_{\alpha \nu}$ K</th>
<th>$T_{\alpha \nu} / T_{\alpha}$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_{\alpha + 2\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>75% O, 25% N₂</td>
<td>50</td>
<td>1,315</td>
<td>1,887</td>
<td>0.70</td>
<td>0.29</td>
<td>0.81</td>
<td>1.91</td>
</tr>
<tr>
<td>75% O, 25% N₂</td>
<td>100</td>
<td>2,069</td>
<td>4,528</td>
<td>0.46</td>
<td>0.25</td>
<td>0.83</td>
<td>1.91</td>
</tr>
<tr>
<td>75% O, 25% N₂</td>
<td>150</td>
<td>3,222</td>
<td>8,940</td>
<td>0.36</td>
<td>0.24</td>
<td>0.84</td>
<td>1.92</td>
</tr>
<tr>
<td>50% O, 50% N₂</td>
<td>50</td>
<td>1,438</td>
<td>1,935</td>
<td>0.74</td>
<td>0.36</td>
<td>0.76</td>
<td>1.88</td>
</tr>
<tr>
<td>50% O, 50% N₂</td>
<td>100</td>
<td>2,518</td>
<td>4,660</td>
<td>0.54</td>
<td>0.32</td>
<td>0.77</td>
<td>1.86</td>
</tr>
<tr>
<td>50% O, 50% N₂</td>
<td>150</td>
<td>4,237</td>
<td>9,177</td>
<td>0.46</td>
<td>0.31</td>
<td>0.79</td>
<td>1.89</td>
</tr>
<tr>
<td>75% O, 25% N₂</td>
<td>50</td>
<td>1,617</td>
<td>2,023</td>
<td>0.80</td>
<td>0.43</td>
<td>0.72</td>
<td>1.87</td>
</tr>
<tr>
<td>75% O, 25% N₂</td>
<td>100</td>
<td>3,233</td>
<td>4,950</td>
<td>0.65</td>
<td>0.40</td>
<td>0.72</td>
<td>1.84</td>
</tr>
</tbody>
</table>

TABLE 2h. Values of Ion Temperatures and $\beta$ for $N_2^+$ Colliding With $N_2$ Using the Mixed Collision Model With $E_{\alpha} = 100$ mV/m

<table>
<thead>
<tr>
<th>Chemical Lifetime</th>
<th>$T_{\alpha}$ K</th>
<th>$T_{\alpha \nu}$ K</th>
<th>$T_{\alpha \nu} / T_{\alpha}$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_{\alpha + 2\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1,603</td>
<td>7,457</td>
<td>0.215</td>
<td>0.09</td>
<td>0.95</td>
<td>1.99</td>
</tr>
<tr>
<td>4</td>
<td>1,732</td>
<td>7,344</td>
<td>0.236</td>
<td>0.11</td>
<td>0.93</td>
<td>1.98</td>
</tr>
<tr>
<td>10</td>
<td>1,930</td>
<td>7,230</td>
<td>0.267</td>
<td>0.14</td>
<td>0.92</td>
<td>1.97</td>
</tr>
<tr>
<td>15</td>
<td>1,972</td>
<td>7,225</td>
<td>0.273</td>
<td>0.14</td>
<td>0.92</td>
<td>1.98</td>
</tr>
<tr>
<td>20</td>
<td>1,998</td>
<td>7,199</td>
<td>0.278</td>
<td>0.15</td>
<td>0.91</td>
<td>1.97</td>
</tr>
<tr>
<td>25</td>
<td>2,008</td>
<td>7,184</td>
<td>0.280</td>
<td>0.15</td>
<td>0.91</td>
<td>1.97</td>
</tr>
<tr>
<td>50</td>
<td>2,050</td>
<td>7,171</td>
<td>0.286</td>
<td>0.15</td>
<td>0.91</td>
<td>1.97</td>
</tr>
<tr>
<td>100</td>
<td>2,077</td>
<td>7,162</td>
<td>0.290</td>
<td>0.16</td>
<td>0.91</td>
<td>1.97</td>
</tr>
<tr>
<td>infinite</td>
<td>2,086</td>
<td>7,153</td>
<td>0.292</td>
<td>0.16</td>
<td>0.91</td>
<td>1.97</td>
</tr>
</tbody>
</table>

than 4-12) actually dominate the scattering process. Thus $O^+$ ions colliding with $O$ and $N_2^+$ in an $N_2$ atmosphere give temperatures that are within 1% of the Maxwell molecule balance. Likewise, for $O^+$ colliding with $N_2$ the balance is within 2% of the Maxwell molecule behavior for all the cases that we have studied. For a single neutral constituent the worse deviations from the constant collision frequency behavior are always when we consider NO$^+$ colliding with O, irrespective of the neutral temperature used or of the depth chosen for the minimum in the interaction potential. We also note that the use of (16) to describe the effect of collisions with a mixed neutral background leads to results that deviate from the Monte Carlo derived temperatures by up to 8% (Table 2g). In that latter case our energy-dependent
momentum transfer collision frequencies differ from the tabulated ones in the energy range of interest for the computations shown in Table 2; in particular, this affects the mean mass definition in (16). Finally we note that in all cases the percentage deviation between Monte Carlo derived and Maxwell molecules increases slowly as the electric field strength increases.

The explanation behind the behavior noticed in Table 2 for collisions with single constituents can be traced to the following considerations. First of all, in the case of RCE collisions the main factor leading to a good agreement with (17) is that \( \beta_0 < \beta_1 \), so that \( \beta_1 \) approaches 1 all by itself. We note that in the polarization case the Maxwell molecule model is less accurate either when the ion mass is larger than the neutral mass or when \( \tau \) is small, or when the electric field becomes very large. In all cases the controlling parameter is the ratio \( \alpha = (\mu_{\text{in}} < g >^2 / 2)/\tau \), where \( \mu_{\text{in}} \) is the reduced ion-neutral mass and \( < g > \) a representative relative speed for the situation considered. When the parameter \( \alpha \) exceeds 1, departures from the Maxwell molecule results are observed. In that case the hard-core repulsive force, which no longer produces a constant collision frequency, takes over the collisional process, so that (17) is no longer valid, at least in principle. See St-Maurice and Schunk [1977] or St-Maurice and Hanson [1982] for comprehensive discussions.

**Ion temperature anisotropy results.** The partition parameters presented in Table 2 are also important by themselves for a proper interpretation of data acquired along a particular line of sight because the temperature at an angle exists for a proper interpretation of data acquired along a particular line of sight. In that case the hard-core repulsive force, which no longer produces a constant collision frequency, takes over the collisional process, so that (17) is no longer valid, at least in principle. See St-Maurice and Schunk [1977] or St-Maurice and Hanson [1982] for comprehensive discussions.

**Table 2** shows that for chemical lifetimes that extend beyond 0.02 of the asymptotic limit at which the ion velocity distribution is no longer affected by chemistry. Before making a more detailed comparison, we have to note that for many altitudes of interest the effect of ion-1 collisions should also be included in a computation of the partition parameters. However, it turns out that this can easily be done in theory if the partition coefficient with neutral collisions alone is known [St-Maurice and Hanson, 1982]. Thus, with the numbers shown in Table 2, a comparison with actual data can be made, even in the presence of collisions with other charged particles. With this in mind, we can now compare our results with the work of various authors who have attempted to experimentally determine the partition coefficients in recent papers; these include Perraut et al. (1984a,b), who found \( \beta_\perp \) varying between 0.67 and 0.77 at 300 km, while \( \beta_\parallel \) appeared to vary between 0.23 and 0.36 (the electric field was about 60 mV/m then). Glatthor and Hernandez [1990] performed a similar study and found \( \beta_\perp = 0.82 \pm 0.18 \) at 312 km. Finally, I. W. McCrea et al. (submitted, 1991) determined that \( \beta_\parallel \) increased with altitude, reaching values between 0.22 and 0.35 at 300 km altitude. These values increased by up to about 50% around 400 km, in agreement with theoretical work which predicts this behavior as a result of the increasing influence of ion-1 collisions with altitude.

As explained by McCrea et al. (submitted, 1991), it is difficult to directly compare theoretically determined values with experiments because of the large number of geophysical parameters that affect the results, for example, the temperature and density of the ionized species themselves. Thus, while there is no inherent contradiction between our calculations and experimental results, it seems worthwhile to extend our comparison to the theoretical estimations that were made by St-Maurice and Schunk [1977], which are always used as a benchmark in this sort of work.

A comparison of our values of \( \beta_\parallel \) and \( \beta_\perp \) with the values estimated theoretically by St-Maurice and Schunk [1977] shows that there is a remarkably good agreement between the two. Specifically, for the pure polarization case with \( m_1/m_n = 2 \), the theory predicted \( \beta_\parallel = 0.53 \) and \( \beta_\perp = 0.74 \). The results obtained for \( \beta_\parallel \) and \( \beta_\perp \) using the present Monte Carlo calculations with NO+ colliding with O and \( E_\perp = 50 \) mV/m give 0.49 and 0.75 for the two values of \( \beta \), respectively. We stress again, that as the electric field increases, however, the ion-neutral collision frequency becomes speed dependent, leading to a smaller value for the Monte Carlo determination of \( \beta_\perp \) while the value \( \beta_\parallel + 2\beta_\perp \) also starts to depart from 2.

For pure polarization interactions with \( m_1/m_n = 0.5 \), St-Maurice and Schunk estimated that \( \beta_\parallel \) and \( \beta_\perp \) should be 0.59 and 0.71, respectively. The Monte Carlo results listed in Table 2 for O+ on N2 produce \( \beta_\parallel \) and \( \beta_\perp \) values of 0.57 and 0.70, respectively, regardless of electric field strength. This is again very close to the earlier theoretical estimates.

St-Maurice and Schunk [1977] also studied two models for interactions that involved a resonant charge exchange. Their so-called "model A" produced \( \beta_\parallel \) and \( \beta_\perp \) equal to 0.34 and 0.83, respectively. However, the Monte Carlo results for a mixed model of O+ colliding with O yield \( \beta_\parallel \) and \( \beta_\perp \) values of 0.24 and 0.88 for \( E_\perp = 50 \) mV/m, with \( \beta_\perp \) decreasing to 0.17 as the electric field strength increases. Our current results therefore exhibit significantly more anisotropy than the "model A" predictions. On the other hand, a comparison with St-Maurice and Schunk's "model B" (\( \beta_\parallel = 0.24 \) and \( \beta_\perp = 0.88 \)) shows near perfect agreement with our smaller electric field calculations. Our calculations therefore indicate that St-Maurice and Schunk underestimated the role played by pure RCE collisions in their calculations. This may well be because we have assumed here that, for reasons of symmetry, the instantaneous RCE cross section must first
be made twice as large as the experimentally determined value in order to account for the fact that for half of the time the electron returns to the neutral atom or molecule itself rather than to the ion. Even though half of our RCE collisions therefore do not take place, the fact that they occur with a larger cross section when they take place does increase their relative importance at a given energy. We surmise that this is why we find a much better agreement with St-Maurice and Schunk's model B, which was described as being a more appropriate model to use for $T_i$ exceeding 3000 K. The fact is that we find that we should already use this value when $T_i = 1500$ K.

A note on pure polarization anisotropies. We end this section by noting that from a more superficial vantage point the ratio $T_A/T_B$ can be viewed as providing a simple quantitative estimate of the anisotropy of the distribution. From that point of view the further the temperature ratio departs from 1, the more anisotropic and non-Maxwellian the distribution is. We then notice that although both $T_A$ and $T_B$ continue to increase with increasing electric field strength for both NO$^+$ colliding with O and the O$^+$ colliding with N$_2$ (Table 2), the temperature ratio also approaches constant values, suggesting, in agreement with the conclusion reached with the log plots, that the non-Maxwellian character of the velocity distribution has stopped changing. This can be contrasted with the mixed collision model situation (O$^+$ colliding with O), for which the $T_A/T_B$ ratio continually increases with electric field strength, while the shape of the distribution also continues to evolve.

3.3. Comparisons With Equivalent bi-Maxwellians

Another useful way to present our results is to display the ion velocity distributions in the form of a ratio between the simulated distribution and a bi-Maxwellian ($f_{BM}$) having the same parallel and perpendicular temperatures. This presentation is useful if the departures from a bi-Maxwellian are not too considerable. If that is the case, this kind of presentation is more quantitative than the log plots; it also has the added advantage that it facilitates the comparison with results from theoretical work which are often expressed in this manner. In these comparisons the chosen bi-Maxwellians are simply the product of two Maxwellians, one in the parallel and the other in the perpendicular direction to the geomagnetic field, each having the same temperatures as the final distribution [e.g., St.-Maurice and Schunk, 1977]. In what follows, the figures thus obtained are labeled as "ratio" plots.

Since we have focused our present study on rather large electric field strengths, we only present ratio plots here for the case of NO$^+$ colliding with O. The charge exchange cases are simply too different from a bi-Maxwellian description here to warrant a meaningful comparison. The bi-Maxwellian comparison does work well for ions and neutrals of unlike masses. In the NO$^+$ case this is mostly because we observe no further evolution in the distribution function beyond electric field strengths of the order of 100 to 150 mV/m, in agreement with the tentative conclusions that we had reached with the log plots presentation.

More specifically, Figure 7 shows the ratio plot for electric fields of 50 mV/m and 100 mV/m. The character of the distribution is clearly changing between these two cases; we find a decrease in the value of the ratio from 0.96 to 0.82 in the center of the ratio plot (numbers not shown in the plot) when increasing the electric field by 50 mV/m while at the same time the ratio increases from 1.09 to 1.31 in the center of the region ($v_A = 1.5v_{thi}, v_B = 0$). There is still some evolution between the 100-mV/m and 150-mV/m cases, although not a very large one. As a case in point, we show in Figure 8 that a 50-mV/m increase in the electric field from 250 mV/m to 300 mV/m produces a practically negligible change in the ratio plots. The numbers change from 1.57 to 1.53 in the center of the region ($v_A = 1.5v_{thi}, v_B = 0$) while there is no change at all in the center of the distribution. This confirms that the distribution reaches an asymptotic shape for some reasonably large electric field value. Furthermore, the ratios that we obtain are small enough that a theoretical description based on a series expansion about a zeroth-order bi-Maxwellian shape should be very likely to produce valid results with this particular type of collision. Indeed, the values shown on the ratio plots presented here compare favorably with earlier theoretical work on the subject. Specifically, the results obtained by St-Maurice and Schunk [1977] are very similar to ours for low to moderate electric field strengths. As the electric field approaches 150 mV/m, on the other hand, we notice that the theoretical model is less...
Fig. 8. Contours of log($f_i/f_{BM}$) for NO$^+$/O$^-$ collisions with $E_\perp$ values of (a) 250 mV/m and (b) 300 mV/m.

depressed than in our calculations in the central regions and is also less enhanced in the region ($v_\perp = 1.5v_{thi}, v_\parallel = 0$). The results obtained by Hubert [1983] are similarly in good agreement with our present results at lower electric field strengths. In contrast with our comparison with St-Maurice and Schunk’s work, however, as the electric field strength increases, the central region in Hubert’s calculations is more depressed than in the Monte Carlo results, while by the same token the region ($v_\perp = 1.5v_{thi}, v_\parallel = 0$) is not enhanced by as much as the Monte Carlo results. It is also interesting to note that Hubert’s [1983] results did produce a saturation of the distribution function for electric field strengths in excess of 150 mV/m, in agreement with our results.

4. ONE-DIMENSIONAL FITTING TECHNIQUES

For distributions that deviate more from a two-temperature, bi-Maxwellian type of shape, we have made comparisons between our results and toroidal types of distributions. Rather than making two-dimensional comparisons per se, however, we have chosen to express our results in terms of various one-dimensional cuts, as most measurements are concerned with one-dimensional ion velocity distributions and not the full three-dimensional description. In the spirit of the work by Kikuchi et al. [1989], we have explored reasonably simple descriptions of the one-dimensional double hump produced by a toroidal type of distribution presented in the log plots of Figures 1 to 6. Using minimization techniques we then produced estimates for the numerical values of the parameters that would best describe the double-humped shape within the mathematical model used. As should become obvious from our presentation, this technique has the added advantage that it provides a quick (though not always accurate) quantitative description of the shape of the ion velocity distribution in any and all directions.

To be more precise, we compared one-dimensional distributions generated by the Monte Carlo computations with two descriptions used in earlier work by Kikuchi et al. [1989]: the so-called “Raman” distribution and the “two-Maxwellian” distribution. Since the statistics of the Monte Carlo distribution are good in the center and quite poor near the edges of the distribution at around 3 $v_{thi}$, we have used a least squares fitting technique that minimized a functional that gave more weight to the regions where the distribution was larger. Specifically, we used a functional $I$ given by

$$I = \frac{1}{N} \sum_{j=1}^{N} [g_i(v_j) - f_i(v_j)]^2$$ (19)

In (19), $f_i$ is the chosen analytical function of $v$ (either Raman or two-Maxwellian), $g_i$ is the distribution calculated using the Monte Carlo technique, and $N$ is the number of $v$ points for which $g_i$ is computed.

To characterize the quality of the fit we have used the quantity $\sigma$ given by

$$\sigma = \frac{\sum_{j=1}^{N} [g_i(v_j) - f_i(v_j)]^2}{\sum_{j=1}^{N} f_i(v_j)^2}$$ (20)

as a measure of the deviation between the Monte Carlo one-dimensional distribution and the analytical fit to it.

4.1. One-Dimensional “Raman” Fit

We first compared the Monte Carlo results with a Raman distribution, namely, a distribution of the form [Raman et al., 1981]

$$g_R(v, T^*, D^*) = \frac{2}{\pi} \int_{-\infty}^{\infty} dv' I_0(2D^*\sqrt{v'^2 + v^2}) \times \exp \left[-D^*\left(r^2 + v^2\right)\right]$$ (21)

where $v' = v/\sqrt{2k_BT^*/m_i}$ and $D^*$ and $T^*$ are chosen so as to minimize equation (19) for any given case. As expected, we have found that the optimal value of $D^*$ decreases as we change the line of sight from perpendicular to B to parallel to B. This is because the distribution parallel to B is always more nearly Maxwellian than in the perpendicular direction (e.g., Figures 1 to 7 as well as previous work on this subject).

Note that we have used (21) to describe the distribution function along any line of sight. This is in contrast with the original use that was made by Raman et al. [1981] and many others after them. In recent work, in particular, Suvanto et al. [1989b] have been using the original Raman description
Unfortunately but perhaps predictably, we have found that the one-dimensional Raman distribution does not always describe the simulated distributions adequately. But, since the quality of a given fit depends on $\sigma$, it is important to have a feeling for the values of $\sigma$ for which a fit should be deemed as "acceptable". For this reason, we have drawn specific examples of fits dealing with different values of $\sigma$ in Figure 9. Thus, Figures 9a–9c show examples of fits that yielded deviations of $\sigma = 1\%$, $\sigma = 5\%$ and $\sigma = 10\%$, respectively, between the Raman and Monte Carlo distributions. First of all, we note that in all cases the Raman fit underestimates the central region of the simulated distributions while overestimating the hump region. This exaggeration of

**Fig. 9.** Examples of (a) $\sigma = 1\%$, (b) $\sigma = 5\%$, and (c) $\sigma = 10\%$ between the Raman fit and the one-dimensional Monte Carlo distribution. In Figure 9a, the aspect angle is 70° for a distribution of NO$^+$ colliding with O and $E_{\perp} = 250$ mV/m. In Figure 9b, the aspect angle is 60° for a distribution of O$^+$ colliding with O and $E_{\perp} = 200$ mV/m. In Figure 9c, the aspect angle is 90° for a distribution of O$^+$ colliding with O and $E_{\perp} = 300$ mV/m.

**Fig. 10.** Optimal $D^*$ values for NO$^+$ colliding with O.

**Fig. 11.** Optimal $D^*$ values for NO$^+$ colliding with O and $E_{\perp} = 100$ mV/m.
the dip and humps becomes quite noticeable when $\sigma$ reaches 5%. The conclusion to draw from this comparison is that fits with values of $\sigma$ approaching 5% should be taken with caution while fits with $\sigma$ values approaching 10% or greater should be considered as having real problems.

Figures 10 through 19 show how the optimal values of $D^*$ vary with aspect angle (the angle between the line of sight and $B$) for various parameters such as electric field, neutral temperature, background neutral composition, and, in the case of $N_2^+$, chemical lifetime. In addition to showing contours of optimal $D^*$ values, several of the figures also contain light and dark shaded regions. In all the figures, regions that are not shaded are regions for which the corresponding fits gave better than a 1% value for the standard deviation defined by $\sigma$ in (20). The light shaded regions are regions for which the standard deviation varied between 1 and 5%, while regions with standard deviations 5% and greater are shown with a dark shade.

Concentrating first on the general quality of the fits (as given by the various shading patterns), we notice that the best results are found for NO$^+$ colliding with O, no matter what the electric field strength (Figure 10) or the neutral temperature (Figure 11) is. Likewise, the case of O$^+$ colliding with $N_2$ gives very good quality fits to the Raman distributions for all electric field strengths (Figure 12), although those fits are of a somewhat lesser quality than the NO$^+$ ones. Other excellent-quality fits are generally obtained only if the electric field is moderate (see the 50-mV/m cases in Figures 14 and 17).
The remaining figures, in the group defined by Figures 10 to 19, indicate that excellent-quality fits can be obtained for fields of the order of 100 mV/m only if the aspect angle is neither near the parallel nor near the perpendicular directions (Figures 15 and 18). This latter feature is also practically always true; namely, intermediate angles tend to give better-quality fits to the one-dimensional Raman description no matter what the collision process is. In most of the cases shown here this seems to be because at large aspect angles the non-Maxwellian features are generally very pronounced and cannot be described by a simple toroidal shape of the kind given by (21) while at small aspect angles the distribution, while not being Maxwellian, is not particularly well described by a toroidal description either (the toroidal description is nevertheless better than a Maxwellian description, as witnessed by the fact that $D^*$ does not really approach 0 at zero aspect angle). This explanation is also consistent with the fact that even when the collision process is closest to the relaxation model situation upon which (21) is based, the fits are not of as good a quality as when a 4-12 interaction is taking place; we notice at the same time, however, that the $D^*$ values are always substantially smaller for the 4-12 interaction or the weak electric field cases, which is consistent with the aspect angle explanation just given.

Turning now to the $D^*$ values themselves, we notice that the largest value taken by $D^*$ is 1 if the interaction is a 4-12
one, irrespective of whether the ion species is O\(^+\) or NO\(^+\). In agreement with the conclusions reached in previous sections we also see from Figures 10 and 12 that \(D^*\) changes only slowly with electric field strength once the field reaches 100 mV/m. This behavior can be contrasted with the charge exchange dominated regime (Figure 13), for which \(D^*\) steadily increases with electric field strength.

Figures 10 to 19 also show that for the 4-12 interaction, \(D^*\) only changes slowly with the neutral temperature (Figure 11). For N\(_2^+\) ions, Figures 14, 15, and 16 also show that the shape does not change very much as a function of chemical lifetime. Nevertheless, there is a small tendency for \(D^*\) to decrease in this case if the chemical lifetime exceeds about 7 collision times (this feature does not stand out too clearly in the figures). A far more important parameter controlling the shape of N\(^+\) distributions, however, is the electric field strength, as can be seen by comparing Figures 14 to 16. This is because of the strong dominance of the pure relaxation model signature in that particular case. Finally, Figures 18 and 19 show that the shape of the O\(^+\) distribution is a rather strong function of the background composition. This should not be too surprising since the O\(^+\)-O collisions are dominated by charge exchange while O\(^+\)-N\(_2\) collisions are dominated by the much weaker 4-12 interactions. Figures 18 and 19 do show that for strong electric field conditions the shape is a smooth function of the background composition and that even when N\(_2\) is only 15\% of the background density, it has an effect on the final shape of the O\(^+\) distribution.

4.2. Two-Maxwellian One-Dimensional Fits

Following the suggestion made by Kikuchi et al. [1989], our Monte Carlo distributions were also compared to a two-Maxwellian distribution having the form

\[
g_M(v, D_M, T^*) = \frac{1}{2} \sqrt{\frac{m_i}{2\pi K_i T^*}} \times \left\{ \exp \left[ -\frac{m_i (v - u)^2}{2 K_i T^*} \right] + \exp \left[ -\frac{m_i (v + u)^2}{2 K_i T^*} \right] \right\}
\]

where \(u = D_M \sqrt{K_i T^* / m_i}\). The advantage of using (22) to approximate the double hump in the one-dimensional distributions is that it is economical since it is a very simple model to use and compute. For practical inversion problems (either incoherent radar spectra or RPA curve fitting), this could have important advantages, even with modern computing capabilities.

Figures 20a-20c show the differences between the best two-Maxwellian fits and the corresponding one-dimensional Monte Carlo distributions for deviations of \(\sigma = 1\%\), \(\sigma = 5\%\), and \(\sigma = 10\%\), respectively. As was the case with the one-dimensional Raman fits, the two-Maxwellian fit underestimates the central region of the simulated distributions while overestimating the humps. Again, this exaggeration of the dip and humps is quite noticeable even when \(\sigma = 5\%\).

As was done in Figures 10 to 19, we have used Figures 21 through 27, rather than a set of tables, to show how the optimal value of \(D_M\) changes as a function of aspect angle and various parameters such as electric field, neutral temperature, background composition, and chemical lifetime in the case of N\(_2^+\). Once again, we have used a shading method to indicate roughly what the quality of the fits was.
A comparison of Figures 10 to 19 versus 21 to 27 indicates two things at a glance. First, the $D_M^*$ are always numerically smaller than $D^*$ values obtained with the Raman distribution (even though the symbols were chosen in such a way that for a given value of $T^*$ and of the line of sight temperature we should have identical values for $D^*$ and $D_M^*$; see Kikuchi et al. [1989] for details). Aside from the somewhat different values of the $D^*$ parameter, a closer look at the shaded regions in the various plots also reveals that the fits given by (22) invariably yield larger values of $\sigma$ compared to fits obtained using (21). The differences become more important as the non-Maxwellian distortions become more pronounced. When the fitting model fails, the two-Maxwellian fits often reach deviations of the order of 20% where the Raman fits are still better than 10% (which, admittedly, is not great but certainly much better).

Aside from numerical differences, all the trends and tendencies that were revealed in Figures 10 to 19 are present in Figures 21 to 27. This means that the model can in fact be used as a quick and useful method to characterize the non-Maxwellian features of the distribution function under moderately disturbed conditions. On the other hand, we must strongly caution the reader about the risks of using (22) to describe the non-Maxwellian features under very strong electric field conditions. The risks would certainly be higher at least than the risk incurred by using (21), as the functional given by (22) would yield very poor minima that would be easily affected by noise in actual data fitting.

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Fig. 21. Optimal $D_M^*$ values for NO$^+$ colliding with O.

Fig. 22. Optimal $D_M^*$ values for NO$^+$ colliding with O and $E_\perp = 100$ mV/m.

Fig. 23. Optimal $D_M^*$ values for O$^+$ colliding with N$_2$.

Fig. 24. Optimal $D_M^*$ values for N$_2^+$ colliding with N$_2$ and $E_\perp = 50$ mV/m.
situations. The use of the functional given by \( (22) \) could then conceivably yield results that could be as bad as using an ordinary Maxwellian in the first place!

5. SUMMARY AND CONCLUSIONS

Owing to the increased research activity bearing on non-Maxwellian ion velocity distributions at high latitudes, we have sought to quantitatively improve the usefulness of existing Monte Carlo calculations of such distributions by using a more sophisticated ion-neutral collision model than in the past. This means that we have paid much attention to the polarization interaction between ions and neutrals, including the case where resonant charge exchange dominates, while also allowing for the full velocity dependence of the cross section. We have also used very large electric fields in our simulations, since it has become evident that such fields do exist and that future theoretical modeling as well as data reduction schemes will have to be able to tackle these large electric field situations. In addition to having an in-depth look at the polarization interaction, we have also added short chemical lifetime effects, to simulate cases where intense fluxes of soft electrons could create large \( N_2^+ \) densities. We have also, for the first time, included a mixed neutral background composition in some of our simulations.

Using our improved polarization collision model for NO\(^+\) colliding with \( O \), as well as for \( O^+ \) colliding with \( N_2 \), we have found that as the electric field increases, the distribution becomes increasingly different from the familiar Maxwellian shape. However, for electric fields reaching about 100 to 150 mV/m, the shape of the polarization-dominated distributions evolves increasingly slowly and basically becomes “frozen”. This is the case even though both the parallel and perpendicular ion temperatures continue to increase; however, the temperature ratio and the overall shape (as given, say, by our \( D^* \) fits) are both becoming very nearly constant. Furthermore, the departures from either the bi-Maxwellian shape or the simple toroidal shape are small enough in the polarization-dominated cases that there should be an excellent chance that analytical schemes based on either a zeroth-order bi-Maxwellian or a zeroth-order weakly toroidal description should converge for any electric field strength.

When resonant charge exchange collisions dominate the collision process, the situation is different. This is the case either for \( O^+ \) colliding with a background gas containing a significant amount of atomic oxygen or for \( N_2^+ \) ions colliding with \( N_2 \). Any interaction which includes a significant amount of resonant charge exchange continues to evolve with increasing electric field strength. Furthermore, when the deviations become large (electric fields beyond 100 mV/m),

![Fig. 25. Optimal \( D_M^* \) values for \( N_2^+ \) colliding with \( N_2 \) and \( E_\perp = 100 \) mV/m.]

![Fig. 26. Optimal \( D_M^* \) values for \( O^+ \) colliding with a mixed \( N_2 \) and \( O \) background and \( E_\perp = 50 \) mV/m.]

![Fig. 27. Optimal \( D_M^* \) values for \( O^+ \) colliding with a mixed \( N_2 \) and \( O \) background and \( E_\perp = 100 \) mV/m.]
they can no longer satisfactorily be fitted to a Raman or a two-Maxwellian distribution, contrary to what Kikuchi et al. [1989] had hoped. In such cases the only hope for analytical calculations seems to be with the use of orthogonal polynomial expansions based on the zeroth-order toroidal descriptions that have been suggested by Hubert [1983]. However, our fits to the commonly used toroidal description indicate that even these expansion schemes might break down as the electric field increases, and, in fact, Barakat and Hubert [1990] showed more conclusively that the expansion breaks down at $E > 200 \text{ mV/m}$.

The expectation has traditionally been that $O^+$ colliding with $O$ would be the most non-Maxwellian distribution one could observe. While this is true in general, we also notice that a rare observation with the EISCAT radar has shown a case where the molecular ions around 250 km had a distribution that deviated more from a Maxwellian shape than any measurement had ever reported [Farmer et al., 1988]. However, under very strong soft electron precipitation events the dominant molecular ion around 250 km could in fact be $N_2^+$ (P. Richards, private communication, 1988). The $N_2^+$ ions undergo "4-12" interactions with atomic oxygen. However, at 250 km they would be produced in regions where $N_2$ dominates, so that resonant charge exchange collision processes would dominate the interactions with the neutral gas. In addition, the $N_2^+$ chemical lifetime is short; the effects due to a short chemical lifetime can be simulated rather well with a relaxation collision model, which is akin to introducing a "pure" resonant charge exchange interaction to take account of the effects due to chemistry. When we included this in our simulations, the result was indeed to produce the most extreme distortions from a Maxwellian distribution for any of our simulations, provided the chemical lifetime was less than about seven elastic collision times. We surmise that this was the situation during the observations made by Farmer et al. [1988].

As far as other results are concerned, we have found that when $O^+$ was allowed to collide with more than one background neutral species, the presence of even 25% $O$ in this background resulted in noticeable toroidal features in the two-dimensional distribution.

To facilitate the quantitative application of our work, we chose to describe the full anisotropy of the distribution function with a series of one-dimensional cuts along various directions with respect to $B$ and to present these results in the form of contour plots. We used a "Raman" type of description to fit the simulations and a so-called "two-Maxwellian" model. We found that the Raman fitting procedure did slightly to appreciably better than the two-Maxwellian fit depending on conditions. Neither method was found to be very good, however, for fitting Monte Carlo distributions under very strong electric field conditions in the presence of charge exchange dominated conditions. Under these conditions we also observed that the best chances for obtaining a reasonable fit with our models of the distribution function are when the aspect angle is neither along $B$ nor perpendicular to $B$.

While the fitting schemes that we explored have some serious difficulties under extreme conditions in the case of $O^+$ ions in particular, we should also stress that on the other side of the coin, under relatively moderate conditions, or for situations where $NO^+$ ions dominate the ion composition, it should be quite feasible to obtain reasonable fits to the distribution by using either simple toroidal fits [e.g., Lockwood et al., 1989], two-Maxwellian models [Kikuchi et al., 1989] or analytical expansion methods [e.g., Hubert and Lathuillère, 1989; Lathuillère and Hubert, 1989]. This is a useful point to note because for large electric field strengths the $O^+$ ions have a strong tendency to be converted to $NO^+$ ions due to the hot ion chemistry that must necessarily be taking place under these conditions [Schunk et al., 1975, 1976].

At this point, there remains the problem of interpreting the one-dimensional ion temperature in terms of the average ion temperature so as to study the energetics of high-latitude situations. To address this question, we have studied the anisotropy in the ion temperature as well as the behavior of the average ion temperature as a function of the relative drift between ions and neutrals, to find out by how much it deviates from the normally accepted values. We posted our results in Table 2 in terms of both the partition parameters and the perpendicular and parallel ion temperatures themselves. We have found that the mean ion temperature does not deviate by more than 10% from the Maxwell molecule relation that is normally used to obtain a first-order description of ion energetics. In our simulations the deviations from the expected values were largest when the ion mass exceeds the neutral mass by a factor of 2, or when we used a very large electric field strength, or when we dealt with a mixed neutral gas background. In our work we have also provided a range of partition parameters and have shown them to be very strong functions of the interaction model, but not of the electric field strength. We have noted that for resonant charge exchange situations our results indicate that the anisotropy is noticeably larger than what has been used until now; our results correspond to the "model B" values of St-Maurice and Schunk [1977] at small electric field strengths and to more extreme anisotropies as the electric field increases.

We believe that the Monte Carlo calculations presented here provide the best description to date of auroral $F$ region ion velocity distributions in the presence of uniform electric fields, primarily because of the self-consistent handling of the ion-neutral interaction potential. As mentioned in the introduction, the method nevertheless has its limitations. The first obvious limitation is statistical, but not too serious in practice; namely, the Monte Carlo method is virtually unable to produce valid descriptions beyond a couple of ion thermal speeds (the number of collisions required for more accuracy becomes unrealistically large). The second limitation is that the computation can only be as good as the collision model used. While it is quite feasible to use any prescription for the ion-neutral interaction potential, we have limited our presentation here to mixtures of 4-12 potentials and resonant charge exchange process. Furthermore, the latter case was treated in a semiempirical manner. The same can be said of the way we handled the chemical reaction effects; further refinements could be programmed if observations warrant it. There is then the more fundamental limitation arising from the fact that we have not included collisions with other charged particles; this is currently being remedied and will be published in future work, where we will also treat wave-particle interactions for cases for which the distribution function becomes unstable to cyclotron or Post-Rosenbluth waves. Finally, the role played by gradients has never been investigated in the published literature for this kind of problem. This includes the effect of vertical
gradients associated with the change in the vertical pressure across the neutral exobase, where the ion temperature change due to friction takes longer to take effect than the time it takes for direct transport to modify the distribution function through heat conduction or even through pressure waves. Analytical work on this aspect of the problem is just starting now (M. Loranc and J.-P. St-Maurice, manuscript in preparation, 1992). The effects due to horizontal gradients in electric fields or even ion densities or a combination of the two also need to be considered. Again, at this point, simple analytical results will be derived first (E. Winkler and J.-P. St-Maurice, manuscript in preparation, 1991) before using the more general but also less transparent Monte Carlo approach.

APPENDIX: SATURATION OF THE ION VELOCITY DISTRIBUTION AT LARGE ELECTRIC FIELD STRENGTHS

We have seen in this paper that the shape of the ion velocity distribution tends to become "frozen" at very large electric field strengths. In particular, for NO⁺ ions this phenomenon occurs when the electric field approaches or exceeds 150 mV/m.

The saturation of the ion velocity distribution can be observed from either the "log" or "ratio" plots. After a certain electric field strength is reached, neither of these two kinds of plots exhibits noticeable changes in the value or shape of the contours if we increase the electric field strength further. In other words, with a proper scaling of the ion velocities the distributions are basically the same. To understand how this arises, we first consider the fact that $f_i = f_i(v_i, v_D, T_n)$. That is, for a given set of ion and neutral masses and a particular "scalable collision process" (see below for definition), the distribution function as a function of $v_i$ depends on only two input parameters, $v_D$ (the magnitude of the $E \times B$ drift) and $T_n$.

The first reason for the saturation of particular ion velocity distributions is that they have to be functions of nondimensional parameters involving $v_i$, $v_D$, and $T_n$. The only possible building blocks for such nondimensional parameters are $v_i/v_D$, $v_i^2/(2kT_n/m_i)$, and $v_D^2/(2kT_n/m_i)$. This means that if $v_D$ is multiplied by $\lambda$, we would only be able to obtain the same distribution if we also multiplied $v_i$ by $\lambda$ and $T_n$ by $\lambda^2$. This also means that we must have

$$f_i(v_i, \lambda v_D, T_n) = f_i(v_i/\lambda, v_D, T_n/\lambda^2) \quad (A1)$$

A saturation in the shape is therefore possible once $\lambda$ is large enough to justify having $T_n/\lambda^2 \rightarrow 0$, that is to say, $2kT_n/m_i < v_i^2$. In other words, when the electric field is sufficiently large, the neutral temperature becomes too small to affect the distribution function. To be more precise, on scales such that $\sqrt{2kT_n/m_i}$ is much smaller than a typical value for the ion speed, the shape of the distribution can saturate. But, since by definition of $T_n$ the basic unit for a value of $v_i$ is the ion thermal speed, $\sqrt{2kT_i/m_i}$, the implication is that the shape can saturate once the condition $T_i \gg T_n$ is achieved. In principle, with anisotropic temperatures and with contour plots of the kind used in this paper, the condition should more precisely be $T_{ij} \gg T_n$ (something which, incidentally, is impossible to achieve with a pure relaxation model and is therefore very difficult to achieve for collisions dominated by the charge exchange process).

While the above discussion stresses that the effect of $T_n$ on a distribution which is a function of $v_D$ has to become negligible before the shape of a velocity distribution can saturate, it provides a necessary, but not a sufficient, condition for the saturation. This is because the resulting function of $v_i/\lambda$ (or $v_i/v_D$) must also be scalable, or invariant. This can only happen if the collision process itself is scalable. This requirement on the collision cross section is far from being satisfied in general. It refers in part to the collision process itself (e.g., hard sphere versus charge exchange collision), which must remain the same as a function of energy (that the collision process itself affects the shape of the velocity distribution should be very clear from all the work that has been done on this topic over the years). This fact is important to notice because in the present paper we are usually dealing with mixed collision models, for example, an attraction $1/r^4$ potential combined with a $1/r^{12}$ repulsion. In such a case, the $1/r^{12}$ interaction dominates at large energies, while the $1/r^4$ potential dominates at lower energies.

The cross section requirement is therefore obviously not satisfied as long as the dominant scattering mechanism changes with energy. But even for one given scattering process, the differential cross section has to fulfill the condition that it be scalable by itself; this condition is not satisfied in general. The cross section requirement means that when the relative ion-neutral speed before a given collision is multiplied by a factor $\lambda$, the scattering angle must not change. Formally, this means having the differential cross section $\sigma$ obey the condition $\sigma(\lambda v_{in}, \chi) = f(\lambda)\sigma(v_{in}, \chi)$, where $v_{in}$ is the relative speed involved in the collision, $\chi$ is the scattering angle, and $f$ is any function of $\lambda$ only. It is known that pure power laws for the interaction do follow this relation [e.g., Chapman and Cowling, 1970]. Therefore when either the $1/r^4$ or the $1/r^{12}$ potential dominates the cross section in our computations, the cross section does become scalable (it can also be shown that this scaling property is the reason why we observe that the ratio of the parallel to perpendicular ion temperatures stops evolving at high energies in the presence of a scalable potential when the effect of $T_n$ becomes negligible; see for example St-Maurice and Schunk [1977] for details on calculations of this kind).

Thus, in addition to requiring $T_i \gg T_n$, for the distribution function to be able to freeze, the ion thermal speed must also, first, be large enough to exceed the energy at which the transition between two (or more) forms of potential occurs; second, the dominant part of the potential for a high enough energy regime must also be scalable, as is the case for pure power law situations. For the examples dealt with in the present paper in particular, there is a transition between two different forms of power law potentials. This transition energy is given by $\epsilon$ in our "mixed 4-12" models (see Table 1 for representative values of $\epsilon$).

There remains the fact that as the mean relative energy between colliding species increases, the collision frequency changes in general through the function $\lambda f(\lambda)$, even for a scalable interaction model like the pure $1/r^{12}$ potential. In the present problem, however, the collision frequency is assumed to be so small that the condition $\nu_i/\Omega_i < 1$ always prevails. This means that the distribution is then gyrotropic. Other than for this feature, it is easy to show that the shape of the distribution function is not affected by the precise value of $\nu_i$, in this strongly magnetized case [e.g., St-Maurice and Schunk, 1977].
We note that the above arguments were derived through a formal solution to the Boltzmann’s equation for the problem at hand, which can be found, for example, in the \textit{St-Maurice and Schunk} [1977] paper. It seems to us, however, that little new would be gained from a more detailed mathematical discussion in the present case.

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