2003 EISCAT Radar School: Non thermal Scattering Mechanisms

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1. Introduction

When it comes to incoherent radar spectra, one of my favourite quotes is from the review paper by Fejer and Kelley [1980] on ionospheric irregularities and their detection by radar. The introductory quote reads:

“The earth’s ionosphere is, in many respects, a remarkably quiescent plasma. The very fact that incoherent scatter radar can often be used to study the ionosphere shows that the only perturbations in the medium are due to thermal fluctuations...”

That paper was written in 1980. Since then, we have uncovered many examples in which the plasma turned out, in fact, not to be as quiescent as we had imagined. As a result the ‘perturbations in the medium’ have turned out not to be as often due to thermal fluctuations as we have previously thought. At the very least, the fluctuations have not been associated with as nearly a perfect state of equilibrium as we had assumed we could use to describe the plasma. This has not necessarily meant that incoherent radar spectra have been rendered useless. It has, however, made life more complicated, but, possibly also, more interesting.

This lecture therefore takes off where the first lecture by Prof Hagfors left us. In Prof Hagfors’ lecture we learned about the spectra that can be expected when the plasma is in near perfect thermal equilibrium. Here we explore what happens when departures from equilibrium are too large to allow for the indiscriminate use of the spectral description presented by Prof Hagfors.

There are two kinds of situations to consider. In the first case, the plasma is not in thermal equilibrium, but the plasma remains stable. That is to say, the wave spectrum differs from the standard description, but it can be recomputed using a standard procedure as long as the velocity distributions of ions and electrons can somehow be determined from our knowledge of the disrupting influences. In the second case, the departures from equilibrium are so large that the plasma waves that we use to study the spectra have become linearly unstable. In that case, positive feedback mechanisms to be described by Prof Forme in a subsequent lecture lead to much more unpredictable situations. The reason for this is that we now need nonlinear theories to describe how the waves saturate, and we are not as good at describing such processes as we are with the linear, near-equilibrium, situations.


Well before plasma waves become linearly unstable (i.e. well before they are predicted by linear theory to ‘grow without bounds’), the velocity distributions of both ions and electrons depart from the equilibrium shape used by Prof Hagfors to determine the spectral shapes that will be observed by incoherent scatter radars. We must therefore be able to account for the departures from the simple Maxwellian velocity distributions of both ions and electrons if we are to properly interpret the spectra collected in the scatter experiments.

Fortunately, the spectral part of the calculation is actually straightforward, because all we have to do is follow the recipe that was introduced in the first lecture by Prof Hagfors. That is to say, all we have to do is compute the integrations associated with the expressions already presented earlier by Prof Hagfors. For this lecture, we will assume singly ionized species and stay above 105 km altitude, therefore neglecting the effect of collisions. This means that with the notation that we will use in the present lecture (borrowed from Sheffield [1975]), we will calculate the spectral density function \( S(k, \omega) \) given by

\[
S(k, \omega) = \frac{2\pi}{k} \left[ 1 - \frac{G_{e}}{\epsilon} \right]^{2} f_{e0} \left( \frac{\omega}{k} \right) + \frac{2\pi}{k} \left| \frac{G_{e}}{\epsilon} \right|^{2} f_{i0} \left( \frac{\omega}{k} \right)
\]

where the frequency \( \omega \) is actually the Doppler-shifted frequency. Similarly, \( k \) is the scattered wavevector. Here also, \( f_{j0} \) is the one-dimensional velocity distribution along the line-of-sight for species \( j \) while

\[
\epsilon \left( \frac{\omega}{k} \right) = 1 + G_{e} \left( \frac{\omega}{k} \right) + G_{i} \left( \frac{\omega}{k} \right)
\]

involves the susceptibilities, themselves given by

\[
G_{j} \left( \frac{\omega}{k} \right) = \int_{-\infty}^{+\infty} d\mathbf{v} \frac{4\pi e^{2} m_{j}}{m_{j} k^{2}} \frac{\mathbf{k} \cdot \partial f_{j0}/\partial \mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v} - i\gamma}
\]
where $\gamma$ is asymptotically small but is needed in order to maintain causality.

### 2.1. Non-maxwellian effects on the ‘ion line’

As Prof Hagfors clearly demonstrated, the so-called ‘ion line’ is by far the most studied and most useful feature of incoherent scatter spectra. This region of the spectrum is controlled by the $f_{i0}$ term and the range of Doppler shifted frequencies, $\omega/k$, that it covers is roughly two ion-acoustic speeds, centered about the mean Doppler shift of the plasma. This region is very narrow compared to the scattering caused by the electron term, $f_{e0}$, which of course covers a range of two electron thermal speeds. As far as the ion line is concerned, $f_{e0}$ is therefore evaluated at the mean Doppler shift of the medium, any other variation being negligible.

So, as long as the frequency is not large compared to the ion plasma frequency, and as long as $T_e/m_i \gg T_i/m_e$ (which is easily achieved in practice in the ionosphere), we can write for the ion line

$$S_i(k, \omega) \approx \frac{2\pi}{k} \left| 1 - \frac{G_e}{\epsilon} f_{e0}(0) + \frac{2\pi}{k} \frac{G_e}{\epsilon} f_{e0} \left( \frac{\omega}{k} \right) \right|$$

$$\approx \frac{2\pi}{k} \left| \frac{G_e}{\epsilon} f_{e0} \left( \frac{\omega}{k} \right) \right|$$  \hspace{1cm} (4)

where the zero Doppler-shift has been repositioned to be the mean plasma drift, which is not a problem as long as the ions and electrons undergo the same drift. Also, the approximation on the second line is valid as long as $m_i T_e \gg m_e T_i$, which is always a good bet in the ionosphere.

Clearly from (4) the shape of the electron velocity distribution can still affect the ion line through $G_e$ and also $\epsilon$, still through $G_e$. By contrast, the shape of the ion velocity distribution has a direct impact through the $f_{i0}$ term, and also contributes more indirectly through its effect on the dielectric function $\epsilon$, this via $G_i$. In the next subsection we assume the electrons to be Maxwellian and investigate the situations for which ions are known to have non-Maxwellian velocity distributions. In a later subsection we also discuss the converse, which also can occur in auroral regions: Maxwellian ions but non-Maxwellian electrons.

#### 2.1.1. Ion line in the presence of non-Maxwellian ions and Maxwellian electrons

At high latitudes the ambient electric fields can be so large that the ions can sometimes move at large supersonic speeds compared to the neutral background gas with which they collide. Clearly, we should not expect the ion velocity distribution to be Maxwellian in that case, since Maxwellians are supposed to describe equilibrium conditions, that is, conditions for which the drifts and temperatures of the colliding species are all the same.

**Non-Maxwellian ions in strong homogeneous DC fields.** There has been an abundant number of publications over the years that have focused particularly on the description of the ion velocity distribution...
under strong homogeneous electric field conditions (e.g. Gaimard et al. [1998]; Hubert [1983]; St.-Maurice et al. [1976]; St.-Maurice and Schunk [1977]; Winkler et al. [1992]; St.-Maurice et al. [1994], and references therein, etc...). Here, we only very briefly summarize the physics associated with this most common departure from the Maxwellian shape. The student will have to go to the published litterature, if interested in the more detailed descriptions and calculations.

In the presence of strong ambient electric fields, non-Maxwellian distributions arise from the fact that collisions constantly try to pull the ions to rest while the Lorentz force constantly accelerates the ions away from rest. This push-and-pull scenario is illustrated more precisely in Figure 1. In Figure 1a we see the velocity space trajectories associated with ions subjected to the Lorentz force, $\mathbf{F} = e\mathbf{E} + e\mathbf{v} \times \mathbf{B}$ when the electric and magnetic fields are both uniform. Notice how the so-called $\mathbf{E} \times \mathbf{B}$ drift is an average that, for many ions, is never an actual velocity, but just an average over a full oscillation cycle. In figure 1b we imagine that one by one the ions are inserted near the origin as a result of a collision with a neutral particle. The ‘collision’ is idealized here and corresponds to a charge exchange between the ion and the neutral. Collisions involving ions in their parent gas (for example O$^+$ ions colliding with O) are reasonably well described by this process, although it turns out that ordinary collisions are also subject to a similar pulling effect, albeit a less dramatic one.

In Figure 2 we illustrate the velocity distribution evolving from the combined action of collisions and the Lorentz force. In figure 2a we simply get a torus resulting from the trajectories seen in Figure 1a. However, in Figure 2b we have increased the ion collision frequency, so that the ions are ‘reset’ near the origin of velocity space before completing a single gyration.

For incoherent scatter radar applications the kinds of distributions posted in Figure 2a are what matter most, as they cover a wide range of ionospheric altitudes (approximately from 130 km to 400 km). Many studies of the ionospheric velocity distributions have therefore been performed for that particular situation. The most detailed and sophisticated numerical studies in this regard have been performed by Winkler et al. [1992] and Gaimard et al. [1998]. The analytical counterpart is basically found in the studies of Hubert [1983] and Hubert and Kinzelin [1992]. Some studies have worried about other complications, such as those introduced by ion-ion collisions [Barghouthi et al., 1993] as well as inhomogeneities in the electric field [St.-Maurice et al., 1994].

Figure 2. (a) Ion velocity distribution in the upper ionosphere, where collisions are rare compared to the cyclotron frequency. The distribution function is shown through shaded contours and is meant to describe a torus in velocity space. (b) In the lower ionosphere, where $\nu_m > \Omega_i$, the velocity distribution now becomes bean-shaped as the ions now undergo a collision well before having completed a full gyration.

The clear detection of velocity distributions of the type shown in Figure 2a was made in situ with a Retarding Potential Analyser instrument on the Atmosphere Explorer C satellite [St.-Maurice et al., 1976].

The kind of one-dimensional velocity distributions associated with the velocity distribution shown in Figure 2a is presented in Figure 3. The parameter $D^*$ shown in the figure describes the degree of distortion from a Maxwellian, with $D^* = 0$ being a Maxwellian and $D^* = 2$ being strongly double-humped, as shown. Numerous analytical, numerical and data studies that we have already quoted have revealed that the shape of the velocity distribution does not go, in practise, beyond a ‘flat-top’ $D^* = 1.4$ shape. The double-humped
Figure 3. Theoretical one-dimensional ion velocity distribution $f_{i0}(v)$ that would be seen when looking at a $55^\circ$ angle with respect to the magnetic field direction under the strong electric field conditions and for the simple collision model that would produce distributions of the type shown in Figure 2a. From Raman et al. [1981].

The shape shown in Figure 3 is therefore not expected, nor is it seen. However, the figure illustrates that the distribution can be still markedly flat near its peak when compared with a Maxwellian with the same one-dimensional temperature and drift.

Non-Maxwellian ions and incoherent radar spectra at large aspect angles. The ion velocity distributions that we have just described very briefly are so strongly anisotropic that, with time, it was uncovered that there were two rather different ways by which they impacted incoherent scatter radar data. The first condition is met when looking at a large oblique angle with respect to the magnetic field direction (typically, 60 degrees or more). Under strong electric field conditions, the one-dimensional velocity distribution can then take the ‘flat-top’ shape discussed along with Figure 3. With $G_e$ evaluated at zero frequency in (4), the spectrum is affected by the non-Maxwellian shapes through three separate factors. First, there is the one-dimensional velocity distribution itself, which is smaller at zero frequency than the equivalent Maxwellian. This by itself would decrease the spectral density at the center and enhance it in the vicinity of the ion thermal speed.

However, as figure (3) illustrates, the final spectrum shows quite the opposite, namely, an enhancement at zero frequency. This is produced by a decrease near the central frequency in $\epsilon$ in the denominator of (4), where $\alpha = 1/k\lambda_D$ and $\lambda_D$ is the Debye length, while $x_i = \omega/kV_{thi}$ and $V_{thi} = \sqrt{2T_i/m_i}$.

The symbols $R_G$ and $I_G$ in (5) stand for the real and imaginary parts of $G_i(x_i)$, respectively. From (3) we have

$$G_i(x_i) = \frac{\omega^2}{k^2} P \int \frac{\partial f_{i0}/\partial p}{\omega/k - p} dp + i\pi \frac{\partial f_{i0}}{\partial p} \bigg|_{p=x_i}$$

where $P$ stands for the ‘principal value’ integral. With a ‘flat-top’ distribution of the kind shown for $D* = 1.5$ in Figure (3) one can see from (6) that the amount of Landau damping provided by the imaginary part of $G_i$, $I_G$, tends to vanish by comparison with the equivalent Maxwellian with a same temperature. This in turn contributes to a reduction in the $|\epsilon|$, as seen by (5). Likewise, one can also see from (6) that the smaller value of the derivative of the distribution near the peak also contributes to a reduction in $R_G$. This being stated, it becomes clear that when the ratio $T_e/T_i$ is increased, the contrast between the central region of
the spectrum and the rest is simply enhanced. That is to say, the zero frequency region in $|\epsilon|$ increases by comparison to the rest, because of the multiplication factor associated with $T_e/T_i$ when $R_G$ and $I_G$ depart from zero in other regions of frequency space.

Therefore, under the normal ionospheric situations for which we expect a double humped spectra because $T_e > T_i$ we then get instead a central peak in the spectrum when the electric field becomes large. This is nicely demonstrated by the observations made by Lockwood et al. [1987] and which are reproduced in Figure (5). Here we see that when the ion drift was small the spectrum had a normal double-humped appearance. However, as the electric field (and the ion drift) increased, a central peak progressively took over the central part of the spectrum owing to the strong reduction in Landau damping.

There have been many systematic studies of the effect of non-Maxwellian flat top velocity distributions both theoretically, numerically, and many observational studies as well. Most notably, Raman et al. [1981] were the first to study the effect systematically. They concluded, as just discussed here, that the non-Maxwellian spectral signatures were affecting the interpretation of the $T_e/T_i$ ratio by introducing a peak in the middle of the spectrum where an ion Maxwellian velocity distribution with the same temperature and drift would produce a minimum. Much later, the flat-top studies were later pursued by Kikuchi et al. [1989] who used Monte Carlo calculations of the velocity distribution. Finally, at around the same time, Suvanto et al. [1989] developed an actual spectral fitting algorithm to analyse the spectral shapes of strongly distorted spectra which proved capable of extracting the shape of the ion velocity distribution and the various plasma parameters whenever the electric field was greater than 50 mV/m and the aspect angle was larger than about 50°.

**The effect of non-Maxwellian ions on small aspect angle spectra.** The above notwithstanding, most of the radar data turns out to come from much smaller aspect angles than what had been used by Lockwood et al. [1987]. In fact, many experiments using the Tromso radar were conducted with the Tromso radar looking along the magnetic field direction. When this was not the case, the angle was often nevertheless not too far from magnetic field alignement whenever the tristatic experiments were conducted. As a result, there has been a second type of non-Maxwellian studies that have required far more care to unravel, because the spectral distortions are far more subtle and nothing as obvious as Figure (5) could be uncovered; basically the one-dimensional velocity distribution is much too close to a Maxwellian when looking in directions approaching the magnetic field direction.

When the angle with respect to the magnetic field (or aspect angle) becomes less than approximately 40°, the shape of the velocity distribution is too close to Maxwellian to produce a spectral signature of its own.
Monte Carlo calculations by Winkler et al. [1992] have shown conclusively that for these smaller aspect angles the distribution rarely departs significantly from the Maxwellian. To be more precise the small aspect angle distributions never deviate from Maxwellian by more than if we had $D^* = 0.5$ under very large aspect angles, no matter what the electric field might be. The only exception would be for a plasma dominated by $O^+$ colliding with $O$. However, under strong electric field conditions, chemistry comes to the rescue and converts much of the $O^+$ ions into $NO^+$ ions because of a huge increase in the $O^+ + N_2$ recombination rate with ion temperature [St.-Maurice and Laneville, 1998]. In practise, therefore, a Maxwellian-based spectral analysis will yield reasonable results at small aspect angles.

In spite of this piece of good news, one must nevertheless keep in mind one very important non-Maxwellian signature when looking at the plasma along the magnetic field lines during strong electric field event. The distribution function remains very strongly anisotropic, with much greater temperatures perpendicular to the magnetic field line than along the field line when the electric field is very large. One must therefore remember that that ‘temperature’ (defined here as the second velocity moment of the distribution function) along the line of sight is related to the parallel and perpendicular temperatures via the equation

$$T_{i,\text{los}} = T_\parallel \cos^2 \phi + T_\perp \sin^2 \phi$$

(7)

where $\phi$ is the aspect angle, $T_\parallel$ is the ion temperature along the magnetic field line, and $T_\perp$ is the ion temperature perpendicular to the magnetic field line. In turn the parallel and perpendicular temperatures are given...
by

\[ T_\parallel = T_n \left[ 1 + \beta_\parallel \frac{m_n}{2kbT_n} \left( \frac{E}{B} \right)^2 \right] \]

\[ T_\perp = T_n \left[ 1 + \beta_\perp \frac{m_n}{2kbT_n} \left( \frac{E}{B} \right)^2 \right] \quad (8) \]

where \( k_b \) is the Boltzmann constant and we are using mks units throughout.

Under the assumption of ions colliding exclusively with neutrals, the parameters \( \beta_\parallel \) and \( \beta_\perp \) have been determined with a Monte Carlo study based on what little available data there is from collision cross-sections from atomic physics experiments. For such a study, Winkler et al. [1992] have found that \( \beta_\parallel \approx 0.48 \) and \( \beta_\perp \approx 0.7 \) for NO\(^+\) colliding with O, creating temperature anisotropies \( T_\parallel/T_\perp \) of the order of 0.75. The situation is far more extreme for O\(^+\) colliding with O, where \( \beta_\parallel \approx 0.25 \) to 0.4 depending on neutral composition (i.e. altitude) while \( \beta_\perp \approx 0.8 \) to 0.72 under the same conditions. For electric fields in excess of 50 mV/m the resulting \( T_\parallel/T_\perp \) anisotropies can vary between 0.35 and 0.70 depending on composition. In “Figures” (6) and (7) I have scanned and reproduced some of the more relevant determinations that were posted in the Winkler et al. [1992] study. It should be emphasized that for people interested in Joule heating and frictional heating studies, the temperature anisotropy corrections that must be applied can be quite significant. In such studies one is normally interested in linking the average ion temperature (i.e. the electric field strength.

Another noteworthy study of the ion temperature anisotropy was from the work of Perraut et al. [1984] who used the tristatic EISCAT observations around Tromso to determine that the line-of-sight ion temperature was increasing with increasing aspect angle when the electric field was strong. Because of the small angles and differences involved, however, this study was a bit tricky, and attempts to improve the study with larger data sets and more refined tools have not been terribly successful. As an illustration of the difficulties involved, Figure (8) offers a reproduction of the anisotropy results obtained by Perraut et al. [1984]. A \( T_\parallel/T_\perp \) anisotropy of the order of 0.70 was uncovered in that study, which is nevertheless consistent with the Monte-Carlo expectations for a plasma dominated by NO\(^+\) ions.

A second type of study of the anisotropy was performed by McCrea et al. [1993] by looking at the ion line spectrum only along the magnetic field line and monitoring \( T_\parallel \) as a function of the electric field strength. This study was important because it clearly established that the ion temperature retrieved along the magnetic field line was, for similar electric fields, systematically smaller that the perpendicular temperature and also systematically smaller than expected from a theoretical computation of the average ion temperature under strong ion frictional heating conditions. In fact, the observations matched the theoretical predictions of the tables shown in Figures (6) and (7) rather well, obtaining values for \( \beta_\parallel \) between 0.2 and 0.4. This represented excellent agreement once allowance for ion-ion and ion-neutral collisions was included in the theoretical computations.

**Other non-Maxwellian ion situations** While the effect of strong DC ambient electric fields has been studied in some detail for the collisional ionospheric plasma below 400 km, there are many more situations for which the ions are not Maxwellian and should affect radar spectra. The trouble with these cases is that not only are they difficult to analyse quantitatively, but they also have too many signatures, making spectral inversions in terms of velocity distributions almost impossible under the circumstances. So, we only mention two cases in passing.

A first example of non-Maxwellian ions that are dif-
Figure 9. Simple calculation of the velocity distributions obtained with the 'evaporative process' when ions get heated by friction during 200 s at a parallel temperature of 5000 K below 400 km. The positions refer to the height above the exobase (400 km) and the times refer to the number of seconds after the heating event was triggered. From Loranc and St.-Maurice [1994].

Figure 10. Observations by the Millstone Hill radar over a field line that had recently been heated by a SAID event. Shown are the altitude profiles of the ion upward velocity ($V$), the ion thermal speed ($V_{thi}$) and the ion acoustic speed ($C_s$), all derived from the observations. From Yeh and Foster [1990].

It is difficult to treat is when frictional heating from the F region triggers an 'evaporation' process [Loranc and St.-Maurice, 1994]. Basically, friction with neutrals terminates around 400 km altitude (for lack of neutrals!) and the hot ions are free to move up. If the electric field has only been large recently one can see the new hot population replacing the old cold one. However, owing to the exponential decrease in density that depends on scale height, the newcomers can find themselves to be in large numbers relative to the background. The results are illustrated in Figure (9). In the initial stages, at high altitudes, they also look like cold fast beams of upwelling ions. Prior to that stage, an observer would see large temperature increases followed by a large parallel motion. This may have been what Yeh and Foster [1990] saw in some Millstone Hill data, which are reproduced here in Figure (10).

While the evaporative distributions have not been studied too vigorously they might play an important role in determining the shape of radar spectra under ion heating situations when the radar looks along the magnetic field direction at altitudes greater than 500 km. Furthermore the process, as illustrated here, could lead to plasma streaming instabilities, which are discussed later in these notes.

One final velocity distribution problem should be mentioned. It concerns the injection of hot gases from rockets or space shuttles in the ionosphere. A first crack at the ion velocity distribution problem has been made by Bernhardt et al. [1998] who chose to describe the ions produced by the charge exchange with the neutral gas very much using the methods presented above for the ions in a strong ambient and homogeneous electric field. The trouble here is that the injection is local in time and space and the complete problem of ion velocity distribution relaxation in time and space is waiting for some
aspiring kinetic theorist to tackle it. Interesting calculations lie ahead, but at least there have been specific experiments set up to study the response of the plasma with the use of an incoherent scatter radar, namely, Arecibo in this case.

Word of caution for those venturing to compute non-Maxwellian spectra. An important assumption used to derive the spectral equations that we have used thus far is that the plasma is stable. When the plasma is stable, thermal-like fluctuations are stimulating the waves which then decay. The incoherent radar spectrum is then made of waves that decay, following their excitation by thermal noise. It can happen, however, that the plasma description is one that is unstable. In that case the eigenfrequencies of $\epsilon$ have positive instead of negative imaginary parts. These eigenfrequencies should be checked before embarking on a comprehensive set of possibly meaningless calculations. For example, we have shown a distribution that was unstable in Figure (3) when the parameter $D_\ast$ reached a value of 2 and the one-dimensional velocity distribution was actually potentially unstable. One way to view this is that in this case Landau damping has been replaced by Landau growth. In a sense the two humps in the velocity distribution correspond to a streaming instability.

2.1.2. Ion line in the presence of Maxwellian ions and non-Maxwellian electrons Fortunately in the great majority of auroral and high latitude situations the electric field is either small enough (less than 50 mV/m) or the line-of-sight close enough to the magnetic field direction, that an analysis based on the standard Maxwellian assumption will provide reasonable results (with the caveat that the line-of-sight ion temperature should be corrected for the strong anisotropies occurring during strong field conditions).

Still, one other situation needs to be investigated, particularly during electron precipitation events and also during strong electron heating events occurring in the lower E region during strong electric field conditions. In those cases the electrons can deviate from the Maxwellian shape in a way that can affect the interpretation of the ion line. We can at least get some insights into the problem by assuming that the electrons are made of a superposition of two maxwellians, one hotter than the other. Certainly this is the simplest way to describe what happens in the presence of electron precipitation. Recalling that the ion line is described by (4) we want to study the effect of $G_e(\omega/k)$ in the vicinity of zero frequency. But we know from (3) that $G_e$ is given by

$$G_e = \frac{\omega^2_{pe}}{k} \int \frac{dv_{ex}}{\omega/k - v_{ex}} \frac{df_{0e}}{dv_{ex}}$$

(9)

In view of the large electron thermal speed compared to the ion thermal speed, we are only interested in the real part of this expression at zero frequency, namely, in the principal value near zero frequency (the imaginary part vanishes at the peak of the distribution). This means having to evaluate

$$G_e \approx -\frac{2\omega^2_{pe}}{k} \int \frac{df_{0e}}{dv_{ex}} dv_{ex}$$

(10)

For purely maxwellian electrons the distribution function is

$$f_{0e} = \sqrt{\frac{m_e}{2\pi T_e}} \exp\left(-\frac{m v_{ex}^2}{2T_e}\right)$$

so that the integral trivially gives the result

$$G_e = \frac{m_e \omega^2_{pe}}{T_e}$$

(11)

The central point here is that the only way the electron distribution affects the ion line is through $G_e(0)$ which in turn depends on $T_e$ through the integral just performed. The integral happens to give $1/T_e$ for a pure Maxwellian. However, for non-Maxwellian, even simple non-Maxwellians made of a combination of two Maxwellian populations (such as a hot population co-existing with a cold one), we have to be more careful because the integral given by (10) is not the second velocity moment of the distribution, which we come to associate with temperature. To be more precise what we mean by ‘temperature’ away from thermal equilibrium situations comes from the computation of

$$3n_e k_b T_e = \int m(v - \langle v_e \rangle)^2 F_{e0} d^3 v$$

(12)

where $F_{e0}$ is the three-dimensional velocity distribution, by contrast with $f_{e0}$ which we have defined as the one-dimensional distribution function. That is, $f_{e0}$ is the integral of $F_{e0}$ over the two velocity components that are perpendicular to the line of sight.

Clearly, for pure Maxwellian situations the result we get from (10) gives the real electron temperature. However, even with the superposition of two Maxwellians with densities $n_h$ and $n_c$, for the hot and cold populations with temperatures $T_h$ and $T_c$, respectively, we get a $T_{eRadar}$ that is not equal to the actual temperature. This can be seen from the fact that for the mixture of
two maxwellians the actual temperature from the second moment of the velocity distribution will give the simple result

\[ T_e = \frac{n_c T_c + n_h T_h}{n_c + n_h} \]  

(13)

whereas the integral from (10) will produce the result

\[ \frac{1}{T_{e,Radar}} = \frac{1}{n_c + n_h} \left( \frac{n_c}{T_c} + \frac{n_h}{T_h} \right) \]  

(14)

This means that the temperature extracted from the radar spectrum is not the real electron temperature in such a case, but rather, it is given by the equation

\[ \frac{T_e}{T_{e,Radar}} = \frac{n_c T_c + n_h T_h}{(n_c + n_h)^2} \left[ \frac{n_c}{T_c} + \frac{n_c}{T_e} \right] \]  

(15)

The result of calculations made for various values of \( n_h/n_c \) and \( T_h/T_c \) are shown in Figure (11). The calculations suggest that one might have to be careful about the extracted electron temperatures during intense precipitation events.

Saito et al. [2000] have explored the same problem by using so-called “Kappa” distributions instead of the simple superposition of Maxwellian distributions discussed here. The Kappa distributions have long energy tails and are therefore qualitatively similar to the distributions just discussed here. One therefore would expect that the electron temperature retrieved with the ion line would be underestimated since the cold population from the vicinity of the peak in the distribution function dominates the integral involved in the calculation of the integral in (10). Saito et al. [2000] have obtained up to 40% underestimations in their calculations.

At any rate, it is important to realize that electrons are never strictly Maxwellian. Small systematic errors should therefore be expected in electron temperatures routinely obtained by incoherent scatter radars. Sometimes in fact, we might face an overestimation in the temperature. For instance, just recently, Milikh and Dimant [2003] have performed an actual computation of electron distribution functions during \( E \) region electron heating events. They proceeded to calculate the temperature corrections implied by the non-Maxwellian situation at hand. In the particular case considered by the authors the distribution was assumed to be spherically symmetric because electrons were only colliding with neutrals in the region of interest. The calculations were made for the strong \( E \) region electron heating events seen at high latitudes whenever the electric field exceeds 50 mV/m. The authors assumed that the electrons were heated by parallel electric fields associated with electrostatic plasma waves a few m or less in size. The waves were produced by the intense Farley-Buneman turbulence known to occur during strong electric field con-
ditions. Irrespective of whether the calculations were terribly accurate or not, the point is that in the best such calculations to date, the authors - whose results are posted in Figure (12) - found the radar temperature to be overestimating the actual temperature by up to 30% in their calculations. Interestingly enough, this is not the kind of result that we can get if we used a superposition of Maxwellians, in which case $T_{\text{Radar}}$ would always be less than the actual temperature $T_e$. This discrepancy seems to be directly related to the fact that in their calculations, Milikh and Dimant [2003] are getting a larger electron velocity distribution slope at small electron energies than they are getting at large energies. This can never happen in a population made of a mixture of two maxwellians.

2.2. Non-Maxwellian distortions and the electron, or plasma, line

It seems that even when things are rather quiet and look boring at first, non-thermal effects still can play an important role. This was seen in a study of the ‘electron’ (or plasma) line published by Kofman et al. [1993]. On the particular day that the study was undertaken there was no magnetic activity and not much structure to speak of, aside from those that were associated with solar radiation. A comparison between the Doppler shifts of the upshifted and downshifted should therefore have revealed nothing of interest. However, there was an important difference between the two. Translated into parallel current densities this difference would have implied auroral currents well above average. The trouble was: there was no auroral currents. The magnetometers were dead quiet and there was no sign of auroral activity anywhere.

A closer look revealed that the difference between the upshifted and downshifted values had to be related to the temperature gradient that showed up in the data. Physically, a temperature gradient is associated with a heat flow. Not only that, but a heat flow is not unlike precipitation in that fast electrons get ahead of the others so that an electric field is created. The bulk electrons then react to that electric field as well. The net result is a skewed distribution with a drift point that does not match the peak in the velocity distribution. The skewness introduces differences between the upshifted and downshifted plasma lines, which are then registered as being different drifts.

To study the problem more quantitatively, recall that as far as the electron line is concerned the ions do not contribute because the Doppler shifted frequency is very large compared to the ion thermal speed. The spectrum for the electron line is therefore given by

$$S(k, \omega) \approx \frac{2\pi}{k} \left| \frac{1}{1 + G_e(k, \omega)} \right|^2 f_{e0}\left(\frac{\omega}{k}\right)$$ (16)

Here we are interested in the Doppler shift. People usually assume that the imaginary part of $G_e$ is so small (the Landau damping term from isotropic electron distributions so small) that we can obtain the Doppler shift by simply finding the real frequency at which the real part of $(1 + G_e)$ is equal to zero. Let’s now suppose that the electrons are Maxwellian except for a heat flow correction term and look at the problem in the limit of high enough frequencies. We have to evaluate

$$\int \frac{dv}{\nu - \nu/k} = -\frac{k}{\omega - k < \nu > z}$$

$$\times \left[ \int \frac{dv}{\nu} \frac{\partial f_e}{\partial \nu} \left( 1 + \sum_n \frac{(\nu - < \nu > z)^n}{(\nu/k - < \nu > z)^n} \right) \right]$$ (17)

Here we have shifted variables to describe everything about the mean drift $< \nu > z$. Then we have assumed $(\omega - k < \nu > z)$ to be greater than $(\nu - < \nu > z)$ and expressed the denominator as a simple geometric series. The next step is to perform an integration by part, using the fact that at $\pm \infty$, $v^m f_e(v) \rightarrow 0$. This leaves us with having to find the roots of the equation

$$0 = 1 + \frac{\omega^2}{k} \sum_{n=1}^{\infty} n \int \frac{dv}{\nu/k - < \nu > z}^{n+1}f_e$$ (18)

Note that the term $n = 1$ does not contribute if $f_e$ is a shifted maxwellian to zeroth order. In any event, the odd terms ($n=1, 3, 5...$) can be summed separately because for a zeroth order Maxwellian they give the well-documented ‘plasma dispersion function’. We will call that function $W$ here, using the notation and definition from Ichimaru [1973]. This means that if we neglect all even moments of the distribution (meaning that we assume that the distribution is not only almost Maxwellian but it is also completely symmetric about its drift point), we then obtain the following equation for the roots from the odd moments the equation

$$1 + \frac{1}{(k\lambda_D)^2} W\left(\frac{\omega - k < \nu > z}{kv_{\text{the}}}ight) = 0$$ (19)

Let us now consider the effect of a heat flow. The heat flow if directly proportional to the $n = 4$ term in the summation in (18). In other words a heat flow
is a measure of the skewness, or of the basic asymmetry of the velocity distribution. Whenever a temperature gradient is present in a gas, the accompanying heat flow introduces a skewness, namely, a fundamental non-Maxwellian signature. To get a leading order correction for the root of the dispersion relation from the heat flow we then simply keep the $W$ function as is and add the one term coming from $n = 4$ to our series. This gives us the following correction to the dispersion relation

$$1 + \frac{1}{(k\lambda D)^2} W \left( \frac{\omega - k \langle v \rangle_z}{k v_{th}} \right)$$

$$-4 \left\{ \frac{\omega_p}{\omega - k \langle v \rangle_z} \right\}^2 \frac{k^3 (2q_e)}{m_e n_e (\omega - k \langle v \rangle_z)^2} = 0$$  (20)

where $q_e$ is the heat flow taking place along the magnetic field. This has been called the heat flow correction to the dispersion relation.

Once the roots are known, there is a standard procedure that allows us to find the upshifted and downshifted lines. Due to relativistic corrections, the up and down wavevectors cannot be assumed to be the same, but the procedure to take such effects into account is well documented. The main point is that once Kofman et al. [1993] used the heat flow correction to a calculation of the roots of the dispersion relation, they got a significant correction that brought them much closer to the observations, but not quite close enough.

Guio et al. [1998] pursued the problem further by introducing a self-consistent description of the velocity distribution. Calculating the electron velocity distribution for conditions associated with those that were observed by Kofman et al. [1993] Guio et al. [1998] went on from there to calculate the spectrum and shifts of the two plasma lines, based on the roots given by (16). Guio et al. [1998] found that the suprathermal electrons did not play too significant a role in the results. However, a calculation that took into account the electric fields produced by the heat flows and their effect on the thermal electrons, agreed quite nicely with the UHF radar data. A comparison of the results for a pure Maxwellian distribution, a heat flow corrected Maxwellian and the numerical results computed by Guio et al. [1998] is reproduced in Figure (13).

### 3. Spectral distortions due to turbulence

When departures from equilibrium in the plasma become too substantial, positive feedback mechanisms usually create very large amplitude waves over a wide spectrum of wavelengths and frequencies. When some of unstable wavelengths reach all the way down to the size which is sampled by the radar, the spectrum cannot in any way, shape or form, be described by anything approaching (1). We will label this the ‘microturbulence’ situation. In addition, there could be turbulence-induced variations that occur over the volume sampled by the radar or during the time interval used to gather the information. This can and often does happen even though the radars are unaffected by microturbulence. We will refer to this second class of echoes as being ‘meso-scale’ turbulence. Finally, turbulence could oc-
cur on larger and slower scales still. This might induce what looks like noise in the retrieved plasma parameters even though the variations might well be ‘real’ in the sense that the plasma had been properly sampled and the variations were in fact genuine. We will obviously refer to this as being ‘large scale turbulence’.

3.1. Microturbulence

Mathematically, though not perhaps not physically, we can understand what happens in microturbulent situations as follows: when sampling the spectrum of waves in the plasma, we are simply looking at a time history of the sampled fluctuations. The waves are excited by known thermal processes and immediately decay. As we sample the plasma, we get a wide spectrum as a result. This is another way to say that the roots to the dispersion relation, namely the equation $\epsilon = 0$, are complex. For these so-called eigenfrequencies, in a stable plasma, the imaginary part is negative, that is, as we said, the waves decay. In an unstable plasma, however, the imaginary part of the frequency is positive. Furthermore, for a marginally stable plasma (waves neither growing nor decaying), the imaginary part is zero. In that case, if we happen to be sampling the eigenfrequency for which the waves are neither stable nor unstable, we are dividing by zero in (1) and getting an infinite amplitude! Obviously, the theory is failing at that point.

Some further insight into the process might be gained if we focus on a particular destabilizing mechanism. We have already noted that the double-humped $D^* = 2$ situation depicted in section 2 might be unstable. This has indeed been shown to be the case by Ott and Farley [1975], St.-Maurice [1975] and Swanto [1989] as long as the wavevectors are very close to perpendicularity to $B$. However, let’s consider something even more straightforward, namely, the change in the wave spectrum that is induced by currents that would be flowing along the magnetic field. In Figure (14) we show the various spectra in the form of a contour plot as a function of frequency and electron drift. We chose to simulate the spectra of a 440 MHz radar with $T_e = 8000 K$ and $T_i = 2000 K$ in an O$^+$ plasma with a density of $10^{11} m^{-3}$.

As Figure (14) shows, for weak enough currents the plasma is stable and the description given by (1) works like a charm. However, as we keep increasing the electron drift and therefore the currents, we are approaching the plasma instability threshold. At that point, we notice an asymmetry emerging in the spectrum, which is still nevertheless well described by (1). In that case, the ion-acoustic peak of the same sign as the triggering electron drift is gaining in amplitude while the other ion-acoustic peak is damped more than it normally would be. If we keep increasing the currents, the asymmetry grows. We can see this by using our spectral expression (1) as long as we make sure that we have not increased the currents to the point that the plasma has become unstable. If we were to go past the marginally stable point and its infinite amplitude, we would be falling into an unstable situation, and the description based on stable plasmas would no longer be valid. However, the spectrum that we would have calculated would have shown no sign being ill-defined. This is something to always be concerned with when doing spectral calculations for non-equilibrium situations. The stability of the plasma must be checked carefully. In practise this is not so hard to do: on should compute $\epsilon$ and check that the imaginary part of its complex root is not positive for the frequencies of interest.

So, when the root of the linear dispersion relation has a positive imaginary part, the predictions made by
linear theory is that amplitudes grow without bounds, which is nonsense of course. In such situations, we have to resort to nonlinear calculations of the spectrum. In other words, the nonlinear terms that were rejected from the linear calculations are taking over the evolution of the waves and they determine what the spectrum should be.

Unfortunately, we have to admit that we are not very good at performing the nonlinear calculations. Partly it is the complexity of the calculations that boggle the mind. Partly too, it’s the number of physical possibilities that suddenly explodes: will the plasma waves saturate by crashing (mode-coupling and wave-steepening)? Will they change the properties of the medium so as to bring the plasma back to threshold instability condition (quasilinear theory)? Will they create secondary instabilities that will take the energy away efficiently? Will inhomogeneities in the medium act on the waves and make their evolution ‘non-local’? Will the nonlinearities be associated with particle fluxes or with nonlinear momentum effects? Or will it simply be that all the mechanisms that we have imagined will compete and work together to produce an unbelievable mess? Obviously the best we can hope for in such situations is to pick one of the dominant mechanisms if we are lucky and there is such a thing. At any rate, when the plasma waves under study are turbulent, we are happy if we can even just identify what creates the waves with the largest amplitude and in the process identify what the dominant frequency will be. Having a proper description of the spectrum is out of the question. Besides, in unstable situations, the spectrum can be highly dynamic and change from one sampling period to the next!

Large amplitude plasma waves are sometimes detected by incoherent scatter radars. This should not be surprising since, after all, many weak power radars are built specifically to observe the large amplitude objects. Those radars are called ‘coherent’ radars as opposed to ‘incoherent’ radars, even though there is no new physics involved, aside from the fact that the waves have a much larger amplitude in the former case.

So, incoherent scatter radars sometimes become coherent radars. The vast majority of the ‘coherent’ echoes is found to be magnetic field-aligned, that is, the structures are elongated along the magnetic field and are detected when looking in a direction very nearly perpendicular to \( \mathbf{B} \). This is the reason why the Tromso radar was built so as to be surrounded by mountains, so that no powerful coherent echo would even come through a side-lobe. By contrast, the Millstone Hill steerable radar antenna is sitting on top of a hill. It consequently frequently observes coherent echoes coming from field-aligned irregularities produced in the E region in the vast majority of cases. Interestingly enough F region coherent echoes are a rarity for 30 cm waves, even in field-aligned situations.

### 3.1.1. Microturbulence in field-aligned irregularities

The vast majority of coherent echoes observed at VHF and UHF frequencies being from field-aligned irregularities, let us start with a brief overview of that process first. The physical reason for field alignment is that electrons are very mobile along the magnetic field direction. Any structure that would not agree to be magnetic field-aligned would quickly be wiped out by the mobile electrons, at least above 95 km altitude. However, the electrons are virtually unable to stop the growth of field-aligned structures because their very strong magnetization forbids them to cross magnetic field lines except to \( \mathbf{E} \times \mathbf{B} \) drift, which does not short out electric fields. Put in another way, the plasma must be driven very hard in order for the irregularities not to be field-aligned (more on this later).

As far as coherent echoes at high latitudes are concerned, we note that while it is near to impossible for the EISCAT Tromso radars to study field-aligned irregularities (papers by Jackel et al. [1997] and references therein are the exceptions that confirm the rule), things have been very different for Millstone Hill. Many coherent echo studies have come out of Millstone Hill. Many coherent echo studies have come out of Millstone Hill, and serious attempts have been made to use the irregularities to learn more about the electric fields that drive them. This, however, is not an easy task for several reasons. First, because they are limited to the E region, the irregularities are located within a relatively narrow latitude band. This makes a vector determination of the irregularity drift a bit tentative for several reasons:

- Very powerful echoes can come in through antenna sidelobes, meaning that care has to be exercised before assessing the direction that the echoes come from. Echo powers up to 90 dB above the thermal incoherent background have been registered, making it virtually impossible at time to be shielded by the antenna beam pattern no matter what the direction of the main beam might be.

- Large longitudinal swipes have to be used to get a vector. The electric field may well not be uniform enough over such a large region so that the direction of the main beam would not point to that reliable even in the absence of other problems.

- The irregularity phase speed is known to saturate
and to increase only very slowly with electric field strength. Repeated studies have confirmed that in the 400 MHz range, saturation does seemingly occur at the ion-acoustic speed $\sqrt{k_b(T_i + T_e)/m_i}$. The increase with electric field strength appears to be related to heating of the E region electrons under strong electric field conditions. Figure (15) was obtained with Millstone Hill data. It shows the electron temperature that was inferred from the speed registered by E region coherent echoes as a function of electric field strength and how this speed matches observations from EISCAT where the electron temperature was measured as a function of electric field strength. In the EISCAT case, this is not hard to do because the radar was looking along the magnetic field direction and the field could be obtained with the tristatic system. In the Millstone Hill case this is trickier: one basically looks to the magnetic east or west, finds events for which there is good reason to believe that the flow follows a magnetic circle of latitude, gets the electric field that way from F region observations and relates the irregularity phase speed to the nearby electric field.

- While the phase speed does saturate, it appears not to be affected as long as the line-of-sight electron drift is less than the ion-acoustic speed. Therefore any drift reconstructed from drastically different directions has to be handled with care because both components must clearly not be saturated in order for an electric field vector to be reconstructed.

- The phase speed (as well as the echo power) appears to depend on the aspect angle. Even though aspect angles are limited to only a few degrees away from perpendicularity this is enough to create interesting changes in the phase speed in response to the ambient electric field.

Nevertheless none of the obstacles are completely insurmountable so that, with patience and care, the irregularities can be used to retrieve at least some information about the strength of the electric field and, in some cases, its direction. However, this is as good as it gets for now. For example, even as simple a property as the scattered power cannot be used to get at the background properties of the medium because it varies too much from one event to the next and with aspect angles and possibly altitude within the 90 to 120 km altitude range not to mention the background density. A good

![Figure 15. E-region electron temperatures versus magnitude of E x B drift, collected from various experiments. The Millstone Hill part was inferred from the saturated velocities of the 30 cm irregularities, unlike the other data which is based on direct electron temperature measurements. From Foster and Erickson [2000].](image-url)

illustration of this is given by a comparison between Figures (16) and (17). In the stronger event the Doppler shifts were large, indicating strong electric fields. However, the power was actually smaller in that case. Foster and Tetenbaum [1991] explored the various factors that could be responsible for these results. The background electron density was weaker, and if so, by how much? Aspect angle effects also needed to be folded in. In the end more research could clearly be done on the subject, although the absence of an independent data source to calibrate the densities makes the researcher’s life very difficult.

As for the spectral width, it certainly cannot be used to determine temperatures in any way, contrary to the incoherent scatter situation. In the coherent case, there seems to be a statistical tendency for spectra to become wider when looking at large angles with respect to the E x B flow. This might be a consequence of mode-coupling. Using a two-dimensional fluid theory Hamza and St.-Maurice [1993] showed that if the instabilities are not driven too hard, mode-coupling in the plane perpendicular to B can be used to saturate the waves.
In that case the mean Doppler shift and the Doppler width are related through the equation

$$\bar{\omega}^2 + \Delta\omega^2 = k^2 c_s^2$$

(21)

where $\bar{\omega}$ is the average frequency, $\Delta\omega^2$ is the square of the spectral width and $c_s$ is the ion-acoustic speed.

In a nutshell, the situation with regard to field-aligned echoes at high latitudes is as follows: (1) the spectra from E region coherent echoes are usually easy to recognize because their power is very strong. Paradoxically, extra care must be taken in very powerful echo situations since sidelobe contamination can affect echoes from any and all directions. (2) The Doppler shift of the echoes can be used to infer the magnitude of the electric field, but not its direction, if the line of sight velocity of the echoes are clearly saturated. When the echoes are clearly not saturated they may sometimes be used to get an electric field measurement or at least one of its components, but only if the aspect angle is small enough. (3) The Doppler width is barely understood and certainly unable to yield any information about temperatures. (4) Similarly, the power varies too much from one event to the next. It should depend on the electric field and the ambient density but other factors like the aspect angle and even the altitude where the echoes are generated seem to be involved.

In spite of space and time limitations forcing us to focus on high latitudes, we should not forget that coherent echoes are also observed by incoherent scatter radars at other latitudes. Most noteworthy is the Jicamarca radar, which only has to look up in order to observe field-aligned irregularities. As a result, the lower E region is frequently blasted by coherent echoes from the equatorial electrojet. Examples of such echoes are provided with the bottom side of Figures (18) and (19). These electrojet echoes have many similarities with their high latitude counterpart, including the saturation effect in the velocities. They tend to be seen at lower altitudes than in the high latitude case because...
km size gradient-drift irregularities seem to greatly help the generation of small scale irregularities and the km size irregularities typically grow below 105 km altitude, at least during the daytime. The small scale equatorial echoes also appear to be remarkably more field-aligned than their high latitude counterpart Kudeki and Farley [1989].

Contrary to the higher latitude radars the Jicamarca radar with its lower frequency is also able to observe field-aligned irregularities with ease throughout the F region. For instance, a region of relatively weak echoes is often observed during the daytime near 150 km altitude, as shown by an example taken from a newly built 30 MHz coherent radar placed at the geomagnetic equator in Brazil in Figure (18). The Doppler shift of these 150 km echoes is believed to provide a very good indication of the plasma drift Kudeki and Fawcett [1993]. Other frequent echo bands are found near the bottom-side of the F region after sunset and at night, around 300 km altitude. Frequently, these echoes evolve to become huge bubbles that are traced with the small scale structures; see Figure (19). The large scale structures are associated with the generalized Rayleigh-Taylor instability, when the nighttime F region plasma is sitting on top of a very low density E region plasma. However, the origin of the small size structures is still a matter of debate even though they are so useful in providing maps of the large stuff.

3.1.2. Microturbulence in more exotic situations: horizontally aligned echoes from auroral structures. On rare but exciting occasions, incoherent scatter radars observe large amplitude ion-acoustic echoes that are not field-aligned. Foster et al. [1988] were the first to report the occurence of high frequency (440 MHz) true coherent echoes (as opposed to satellites) with wave vectors aligned with the magnetic field direction. The echoes were not satellites because they were staying put in spite of moving at km/s speeds and also because they favored a particular morphology, namely: they had a tendency to occur in the so-called mid-latitude trough. In that region the electron density is depleted owing to the absence of precipitation and yet, electric fields can still at times penetrate the region. The echoes seemed to be associated with intense parallel current spikes that would have periodically been firing along the field lines. Figure (20) reproduces one of the key figures from that paper, complete with the original caption.

A unique fact of the observations presented in the [Foster et al., 1988 paper was that, in one of the instances shown, the unstable ion-acoustic peak turned into the ion plasma frequency at some high altitude, and then into the first harmonic of the ion plasma frequency, higher up still. This suggests a rare instance for which the radar was able to probe plasma inside a Debye sphere owing to some very large amplitude ion oscillations. More precisely, the expression for the ion-acoustic waves is given by (e.g. Chen [1984])

$$\omega = \frac{k}{m_e} - \frac{1}{1 + k^2 \lambda_D^2} \frac{m_e}{m_i}$$

where $\lambda_D^2 = \epsilon_0 k_0^2 T_e/(4 e^2)$. When the density becomes small the radar samples structures that become increasingly close to the Debye length $\lambda_D$. Normally, collective interactions have decayed by that time and the signal to noise goes to pots. To have recovered the ion plasma frequency from the echoes required being inside a Debye sphere, i.e. the condition $k^2 \lambda_D^2 >> 1$. Hot electrons probably helped meet that condition, not just small plasma densities.

A few years later, the same phenomenon was detected by Rietveld et al. [1991] using the EISCAT Tromso radar at 933 MHz. In this instance the geometry was rather different in that the radar was observing directly along the geomagnetic field lines whereas Millstone Hill had a more oblique geometry, owing to its position. Once uncovered, the EISCAT results not only confirmed the earlier Millstone Hill findings but added a lot of dynamical and morphological details. Figure (21) has been shown repeatedly but warrants being shown once more because of the richness of the features that were uncovered. A key point is that from one 10 s interval to another, the spectra were changing dramatically, indicating a feature that might operate on time scales less than 1 s. In addition, over the 10 s interval, both ion-acoustic peaks would be enhanced at some heights, but in other time intervals or altitudes only one peak would be enhanced.

Since these discoveries, numerous other publications were presented having for goal to unravel the circumstances behind the trigger of the echoes and, ultimately, understanding their origin. The interested student can consult the recent Sedgemoor-Schelthoss and St.-Maurice [2001] review of the subject for details. What has emerged are what appears to have been two separate explanations for the observations.

The first class of theoretical ideas was essentially proposed in the [Rietveld et al., 1991] who argued that the large amplitude irregularities are produced by a streaming instability involving fast thermal electron drifts. The streaming had to be generated by parallel electric fields generated at ionospheric heights. The thought
at first was that unusually large fluxes of precipitating electrons would be stopped at certain altitudes that depended on their energy and that Pedersen currents would be too slow to remove the charge, thereby creating intense localized parallel electric fields for a few seconds. A first modification of that particular notion was soon introduced, based on other observations by Collis et al. [1991]. In that particular instance, coherent echoes of opposite signs were observed not in the middle of an intense arc but rather on what seemed to be on the edge of the precipitating structure (see the review by Sedgemore-Schelthess and St.-Maurice [2001] for additional points made on this observation). Based on this, St.-Maurice et al. [1996] and Noel et al. [2000] assumed that return currents on the edges of arcs would be a better candidate for at least that class of observations. These authors therefore computed the electrodynamics associated with 100 to 200 m wide cut-offs in precipitation and obtained parallel current densities greater than 500 $\mu$A/m$^2$ in the transition region. Such currents appear, per se, not to be enough to create a two-stream instability. However, Cabrit et al. [1996] have suggested that non-Maxwellian ions might decrease the instability threshold, while the work by Ganguli et al. [1994] also suggests that shears can strongly contribute to decrease the streaming instability thresholds. In other words, calculations based on a pure two-stream calculation have to be taken with a grain of salt when it
comes to actual numbers. Ball park numbers are all we should consider at this point in time.

In the streaming category we should also mention the mechanism proposed by Wahlund et al. [1992] who proposed that 10% H$^+$ ions streaming at the ion-acoustic velocity relative to the rest of the plasma could also trigger the observed instabilities. One thing to keep in mind is that this mechanism should be considered only for echoes found well above 300 km, where there is a sufficient amount of protons and where collisions with neutrals do not hinder their drift.

Not long after the initial proposal by Rietveld et al. [1991], Forme [1993] came up with another explanation involving cascading from Langmuir turbulence to ion-acoustic waves, much as is believed to happen in heating experiments. A classic illustration of the mechanism at work is illustrated in Figure (22) and a discussion of the basic process can be found in various plasma texts (e.g. Chen [1984]). At first, Forme was unable to explain the presence of ion acoustic enhancements of either or both signs that would have been triggered by downgoing precipitating electrons triggering the electron turbulence. However, he later refined the study [Forme, 1999] to show that a double cascade could explain the observations.

As the theoretical ideas have been sorting themselves out, new sophisticated experiments have been performed. The initial hunch created by the Collis et al. [1991] observations that the radar echoes are accompanied by optical signatures has reinvigorated by the findings of Sedgmore-Schulthess et al. [1999] and, most recently, Grydeland et al. [2003], who found the coherent echoes to be triggered in association with discrete optical features over the same field lines, or very close. Still, the fact that the observations are related to optical structure does little to sort out the competing mechanisms because in both instances precipitation is involved. In one case it causes electrodynamical structures that lead to currents carried by thermal electrons or by light ions. In the other, Langmuir turbulence is created first, presumably through precipitating electrons.
We do not know which explanation (streaming as opposed to cascading) is right, primarily because there are convincing examples that sometimes clearly favor one mechanism over the other. For instance the streaming mechanism looks very good when facing the alignment of echoes of a single sign along one precipitation boundary with the matching sign on the other side [Colliis et al., 1991; Foster et al., 1988; Foster, 1990], with little evidence for irregularities in the precipitation regions per se. On the other hand, the double cascading mechanism looks very good when considering simultaneous enhancements of ion-acoustic echoes of both signs [Forme et al., 2001] particularly when new evidence has shown that these echo regions are sometimes within 150 m of one another [Grydeland et al., 2003].

It may therefore well be that the enhanced ion acoustic echoes that are observed along the magnetic field are triggered by more than one or even two mechanisms. After all, the only reason for the trigger of plasma instabilities is the existence of a fundamental departure from thermal equilibrium. It really does not matter what the nature of the departure is, nor what the triggered waves will be. Ion-acoustic waves are preferably observed by incoherent scatter radar and so, if they are destabilized by some process, this is the instability that we will see. Ion-acoustic waves are naturally less damped than other waves in the plasma and so they may be easier to destabilize anyway.

Still, there are situations for which we seem to observe both types of mechanisms, namely: we see strong evidence for simultaneous occurrence of both ion-acoustic enhancements, but above and below that...

**Figure 21.** Stacks of self-normalized spectra obtained at successive 10 s intervals with the EISCAT 933 MHz radar looking along B. Coherent echoes can be seen at several of the ion-acoustic peaks. Note the observed preference for downshifted enhancements higher up and upshifted enhancements lower down, with both peaks enhanced in the middle at times. This preference was later confirmed with a statistical study by Rietveld et al. [1996]. A unique feature of the observation shown here is the world record low altitude for such coherent echoes, namely, 145 km. From Rietveld et al. [1991].
Figure 22. Parallelogram construction showing the $\omega$ and $k$-matching conditions for an electron decay instability that triggers an ion-acoustic mode at the frequency $\omega_s$. In this diagram $\omega_I$ is the incident wave, and $\omega_s$ and $\omega_f$ the decay waves. The straight lines are the dispersion relation for ion waves in the limit of small Debye lengths. The wide parabola describes the dispersion relation of the corresponding relation for electron plasma waves (after Chen [1984]).

height we see an enhancement of only one type of echo, and the signs of the enhancement on each side appear to reverse favoring downshifted peaks higher up. Forme et al. [2001] have proposed that in the presence of the cascading mechanism, density gradients could lead to the gradual replacement of a downshifted echo by an upshifted echo, with a transition region where both lines are enhanced. I propose that something like the following scenario should also be considered to be taking place in such situations:

- From the lower depths of the ionosphere, thermal electrons start to flow in response to a parallel electric field generated at ionospheric heights either because precipitating electrons are stopped before reaching the conducting E region or because a strong conductivity gradient leads to charge accumulation on the edges of the precipitation region.

- Somewhere above 250 km (and, far more rarely, in a deep valley between the F region and the conducting part of the E region) the currents carried by the electrons become essentially one-dimensional. As the total electron density decreases with increasing height, the electron flow increases. At some point the flow becomes large enough to destabilize one of the ion-acoustic peaks directly.

- However, as the density continues to decrease and the electron temperature continues to increase in response to the friction associated with the parallel currents, Langmuir turbulence may or may not ultimately be triggered at the altitudes observed by a high frequency incoherent scatter radar. Whenever it is, then the intense Langmuir turbulence will feed both acoustic peaks in the same place and simultaneously, following the prescription worked out by Forme [1999].

- The space charge associated with intense Langmuir turbulence triggers new events such as ion conics that propel the ions upward through the mirror force, creating current closure at the turbulent heights through an effective ion demagne-
Figure 24. Sandwhich arrangement of coherent echoes reported by Foster [1990]. At high altitudes coherent echoes of the type described in Figure (20) were being detected, while in between the two regions of non-field aligned echoes an unusual example (for 30 cm structures) of F-region field-aligned echoes were being detected. Notice how the sign of the Doppler shift of the echoes suggested the formation of a current triad, much like the kind of triad that could be inferred from the Collis et al. [1991] EISCAT observations.

3.1.3. More weirdness: the Foster perpendicularly-sandwiched echoes from the F region. If one looks at enough data, there is always some fascinating example of new physics that has not been looked at by anybody else. Incoherent radars, with their extreme sensitivity that goes down to thermal levels, certainly offer a great opportunity to study plasma physics in a natural environment and to uncover new phenomena that have not been documented before, this without ever leaving the ground! Of course, great dangers lurk for the would-be explorer who might want to set foot in this jungle. As we will see below there are many traps to avoid, which are related to various experimental artifacts and can lure the observer into discovering things that do not exist! Nevertheless, the very danger may make the trip more fun.

With this strange way to introduce it, I would now like to present the little-known, but highly provocative data set that was unraveled by Foster [1990], and which I have reproduced in Figure (24). Foster [1990] observed coherent echoes with wave vectors along $B$. The non-field-aligned echoes were showing downgoing currents on one field line and upgoing currents on an adjacent field line. This much was not new: we have already shown a similar arrangement of coherent echoes from the work of Collis et al. [1991] and have associated it with two-stream instabilities associated with the intense parallel current densities on the sharp edge of intense precipitation regions. The up- and down-shifts on each side of the arc then reflect the so-called ‘current triad’ that flows through arc regions when the arcs are used to short-circuit a strong ambient electric field.

In order to have a ‘triad’ we require Pedersen currents to be flowing somewhere in the $E$ and/or $F$ region so as to at least attempt to short out the field-aligned currents. This is where the Foster [1990] observations become interesting: in the middle of the F region, sandwiched between the field lines with upper perpendicularly aligned coherent echoes, Foster observed field-aligned coherent echoes at 300 km that looked identical to their lower $E$ region counterpart and appeared to suggest that a Farley-Buneman-like instability was taking place high up in the $F$ region. There are two problems with this: first of all, observations of $F$ region irregularities at 30 cm in the $F$ region are practically non-existent. The physical reason for this should simply be that molecular diffusion is efficient at getting rid of the structures at those heights. To observe large amplitude structures as small as 30 cm in size would therefore require a very strong and unusual direct kind of forcing. Secondly, the ions should have been strongly magnetized in the $F$ region and Hall currents should not have been present to excite an instability in the Hall direction. Nevertheless, here we are: against all odds we find a modified two-stream instability that should not be there at a wavelength that should be way too small for irregularities to be detected.
Obviously, not much has been suggested to explain the riddle raised by the Foster [1990] observations. However, St.-Maurice et al. [1994] pointed out that in the presence of important enough divergences in the electric field the ions should not be considered to be magnetized. In weakly collisional regions such as is found at 300 km in the F region, this turns out to translate into the generation of Hall currents, without having Pedersen currents. Thus, it could be that in the vicinity of the field lines carrying the intense parallel current densities, the ions started to carry unusual Hall currents so that a modified two-stream instability was created by these currents, much in the way it is created in the E region. Even though this means that the echoes were rather localized, the echoes would then have had to be powerful enough to overwhelm the echoes from the incoherent scatter processes coming from the rest of the sandwiched region. This is nothing new. I remind the reader that the data obtained by Grydeland et al. [2003] shows the horizontally aligned echoes to come from a region as narrow as 150 m in size, even though the radar field of view covers 10 to 15 km in the horizontal direction.

3.1.4. Danger lurking: hidden coherent echoes

Cabrit et al. [1996] have reported a rare occurrence of simultaneous UHF and VHF radar profiles at Tromso that clearly disagreed with one another in spite of the fact that both radars were observing the same plasma at the same moment and from basically the same direction. In the event presented in their paper Cabrit et al. [1996] showed that the VHF power was considerably stronger than the UHF power at times. This observation is reproduced in Figure (25). The observation contains several troublesome features that should serve as warnings: first the spectra from the regions of enhanced VHF power show no sign of asymmetry, anywhere. Asymmetries make coherent echoes easy to identify. Not so the symmetric spectra. A second troublesome feature is that, as far as the numbers are concerned, the electron densities from the VHF look realistic, though on the high side. Since there was evidence for precipitation in the area, the enhancement could have been interpreted as real. In other words, were it not for the simultaneous UHF observations, there would have been no obvious way to tell that the VHF power enhancement was due to coherent echoes, aside from the somewhat bizarre power profile.

Short of invoking a strange situation with two similar counterstreaming beams of thermal electrons, a similar enhancement in both ion acoustic peaks throughout the whole region of coherent echoes is not something that should have been expected from any of the theories that have been proposed to date. And yet, an important similarity with other coherent echo events was that the data were recorded during the appearance of strongly elevated electron temperature enhancements in and above the unstable region. The Cabrit et al. [1996] observations therefore represent a challenge for the physics as well as a warning about coherent echoes that may remain hidden by looking very much like incoherent echoes.

3.1.5. More danger: satellites that may look like coherent echoes

Sometimes even hard targets like satellites can play tricks on us. Normally, satellites are easy to identify: they move quickly through the field of view and look nothing like incoherent scatter or even naturally occurring coherent spectra. Complications can arise when the echoes are weaker, that is, when the satellites come through the sidelobes, but these signatures can be sorted out since the Doppler shift will vary rather quickly for nearby orbits.

Another class of satellite echoes can be much trickier to identify however. For this class, satellites positioned 10,000 km away from the radar could be passing through the field of view of its main beam and show up through range aliasing effect (that is, their echoes are...
received not from the pulse being processed, but rather from a previous pulse sequence). If the time between pulse sequences is held fixed, then the echo from the satellite looks like it is coming from an object moving at close range. Also, from the huge distance involved, the satellite motion looks like a straight line and the Doppler shift will hardly vary at all. Finally the power could be comparable to the power level associated with naturally occurring coherent echoes. To add salt to the wound, the Doppler shift of the satellite might even match the ion-acoustic speed.

Rietveld et al. [1996] showed a couple of examples of such range aliased satellites. The fact was that absolutely nothing was going on in the plasma at the time except for this echo that was sustained for a couple of minutes and came back the next day at roughly the same time. It was therefore not too difficult to rule out the echo as a natural phenomenon.

However, in Figure (26) I show another satellite example that we obtained while looking at an interesting storm event. This one did fool us. In this instance the satellite turned out to show up at a moment when there were important changes in many of the plasma properties: the topside of the ionosphere was ‘caving in’, the electric field was undergoing a steep climb, and the plasma parameters underwent a transition from a smoothly varying behavior to a much more turbulent behavior. Add to this that the Doppler shift of the echo and its actual motion were moving at a speed very close to the ion-acoustic speed, and the fact that the echo itself showed up first very near the F region peak. It was therefore very tempting to think that we had observed a boundary in the plasma and that some kind of soliton had been triggered. We had even generated a nonlinear theory for the observations. But the referee of the submitted paper graciously went to work and identified an object 10000 km away that was able to reproduce all the properties of the observations. The Doppler shift and motion of the object matched the observations and the object was recorded by the radar precisely when it was crossing the center of the radar beam.

I learned a dual lesson from the experience. First,
when looking for coincidences, sometimes that’s just what you find: coincidences. Secondly, while trying to document the event I ended up with a storm study that was very interesting in its own right. This suggests that a detailed study of a well documented event can be full of interesting results, even if the original motivation ended up being an illusion!

3.2. Mesoscale turbulence

While turbulence rarely makes it to wavelengths less than 1 m in the F region, scales of the order of 10 m and greater are still frequently involved in large amplitude fluctuations. This allows for example the SuperDARN array of radars [Greenwald et al., 1985] to map out the ionospheric convection using HF radars that probe decimeter size irregularities in the plasma.

It is therefore entirely possible to face situations for which the radar cross section is, by itself, perfectly normal and yet the spectrum is not at all what we would expect. Swartz et al. [1988] brought this up as an alternative to a non-Maxwellian interpretation of the spectral shapes shown in Figure (5). They argued that shears of the right kind could (with some effort I guess) produce similar effects. While their points about shears was correct, they went wrong with the assumption that Maxwellians provided any valid description of the radar cross section. A similar proposal was made by Knudsen et al. [1993], this time regarding ion temperature anisotropies observed with EISCAT. In this case ‘all the authors needed’ was 2 km/s oscillations in the plasma drift, which is a rather extreme of turbulence and requires non-Maxwellian inputs to start with.

But, unfortunately, both the Swartz et al. [1988] and Knudsen et al. [1993] started with the erroneous premise that a Maxwellian is an adequate starting point for the description of the velocity distribution. However, it simply makes no sense to imagine that a supersonic plasma would have nice simple Maxwellian velocity distributions. If anything, the distortions that we get could have been a lot more dramatic, where it not for the fact that the magnetic force brings the ions back to smaller velocities as the physics of the \( \mathbf{E} \times \mathbf{B} \) motion unfolds. In addition, it’s not as if our knowledge of the velocity distributions had stayed qualitative. First of all, on the observational side, direct satellite observations of the effect have been made [St.-Maurice et al., 1976] while on the side of calculations comprehensive Monte Carlo simulations have been used to calculate the distribution functions in support of the data analysis, which includes incoherent scatter radar calculations.

Nevertheless, one point remains: shears (as well as time-dependent phenomena), will affect the radar spectrum. The reason is trivial but important: if the radar beam intercepts a region say 20 km wide by 10 km long, and if turbulence extends down to, say, 100 m scales, the incoherent scatter radar will produce a spectrum that’s a convolution of spectra over a range of differing velocities and even electron densities. If the turbulence in the velocity field is large enough, then clearly the spectrum will broaden and flatten. This will produce two biases in the interpretation: first \( T_i \) will appear to be larger than it actually is and, secondly, the electron temperature will tend to be underestimated because of the flattening of the spectral shape. Swartz et al. [1988] also suggested that spectral asymmetries could arise through the presence of systematic shears, as opposed to just plain turbulence. Mild asymmetries are indeed frequently seen in individual spectra, but are simply ignored. At any rate, it must be remembered that these asymmetries are not the same as the asymmetries discussed in relation to coherent echoes. In that latter case, there is a substantial increase in scattered power as well (see, for example, Figure (14)).

This nevertheless brings us to a gray area with plenty of space for future research. While a radar may well...
be observing a wavelength that is not directly involved in turbulence, the distribution function itself may be affected by said turbulence. For instance if, as suggested by Swartz et al. [1988] and Knudsen et al. [1993], the shears in the plasma are as important as they argue they could be, the ion distribution function will most definitely be affected by the changes in the electric field [St.-Maurice et al., 1994]. An example of a 2-D velocity distribution and corresponding one dimensional velocity distribution in a shear flow region is provided in Figures (27) and (28), respectively. Our group is also looking at the effect of purely temporal variations in the electric field since they too will change the distribution function and, therefore, the radar cross-section.

As a result, mesoscale turbulence does not affect the radar cross section directly (no coherent echo) but it can do so indirectly, not just through a trivial convolution effect, but also by changing the ion velocity distribution and through that, the radar cross-section. The effects on the radar cross section have so far not been investigated because the velocity distribution calculations are still in their infancy. Besides, the number of possibilities increases very quickly when these new parameters are introduced into the calculations. Therefore, without an independent way to monitor at least some of the turbulent properties (using supporting HF radar data perhaps), pretty much any out of the ordinary spectral shape could be explained.

3.3. Larger scale turbulence

Variations can also happen over spatial and temporal scales that are larger or longer than the scales used to sample a radar spectrum. In that case we could observe an increase in the variability of the retrieved parameters that has little to do with mesocales or microscales. In other words, while individual spectra might well have only minor distortions in association with the smaller scales, they might exhibit a large degree of variability from one analysed sample to the next. An example of this sort of thing is shown in Figure (29). Before a strong region of SuperDARN echoes appeared over the EISCAT field of view, the parameters all had a very smooth variations with time. As soon as the SuperDARN region of scattering appeared over EISCAT, the parameters started to show a large amount of variability.

There could have been two different reasons for this: mesoscale phenomena might have affected the spectra and make the analysis more uncertain, thereby causing the increase in the point to point variability. Alternatively, the large scale turbulence was at the origin of the fluctuations, and the variability was not causing major problems with the spectra themselves. Since the spectra were acquired along the magnetic field line large scale turbulence appears to have been more likely; non-
Maxwellian distortions and drift fluctuations are not as easy to come by along the magnetic field direction. Of course, it is entirely possible that both mesoscale and large scale effects were affecting the analysis. After all, cascading means that there is a tendency for mesoscales to be excited in the presence of larger scale fluctuations.

4. Summary, conclusion

Going back to the initial quote for this lecture it does seem truly amazing that we are able to do so well with incoherent scatter radars even when departures from equilibrium are important. We have seen that the lack of equilibrium affects the velocity distributions enough to modify the radar cross sections and the temperature interpretations of the ion line. We have also seen that turbulence can affect the radar spectra at three different levels: it can, sometimes not very obviously, change the radar cross section directly if turbulence reaches down to the scales used by the radar to probe the plasma. It can also change the ion velocity distributions and change the cross sections that way. As well, turbulence introduces a natural convolution of spectra which will create ion and temperature interpretation problems. Finally, turbulence makes itself clearly manifest by creating an increased variance in all the retrieved plasma parameters.

The ultimate irony of all this may well be that we are mostly interested in departures from equilibrium, at least if we study auroral physics or equatorial dynamics. Therefore, the very processes that introduce challenges in our interpretation of the spectra are the processes that we are trying to understand. As a result, there are still very many problems and challenges that are waiting to be tackled. Any taker?

References


