EP 225 Waves, Optics, and Fields

Website: http://physics.usask.ca/~hirose/ep225/
contains

- Course outline
- Laboratory instruction
- Notes
- Past exams
- Animation

Instructor: Akira Hirose   Office Physics 66
akira.hirose@usask.ca
phone: 966 6414

Marks

• Assignments (9 or10)	15%
• Laboratory	15%
• Midterm	20%
• Final	50%

Laboratory

Brian Zulkoskey, Lab Instructor, Room 115 Physics, ph. 6439
brian.zulkoskey@usask.ca

LABS — START OF CLASSES

Students should be advised that labs for EP 225 will begin Monday, January 7. ***For these introductory sessions only, students will meet in the rooms listed below at the indicated times:***

MON 7 JAN 1:30 p.m. L2A & B RM 126 PHYSICS
WED 9 JAN 1:30 p.m. L4A & B RM 129 PHYSICS

- Students should bring a copy of "A Laboratory Manual for Engineering Physics 225.3, REVISED 1997". These are available at the Book Store.
- If a student misses the lab period for which he/she is scheduled, he/she should see Brian Zulkoskey in Room 115 Physics as soon as possible.

Lab Schedule

EP 225 – 2008 LAB SCHEDULE – all labs in Room 113 Physics

Lab Sec\Expt	L11	L13	L16	W2	L12	Make-up Labs
L2A & B (Mon)	Jan 14	Jan 28	Feb 11	Feb 25	Mar 10	Mar 24
L4A & B (Wed)	Jan 16	Jan 30	Feb 13	Feb 27	Mar 12	Mar 26

Lab Introductions: **L2A & B, 1:30 p.m., Monday, 7 January – Rm 126 Physics**

L4A & B, 1:30 p.m., Wednesday, 9 January – Rm 129 Physics

Lab Titles:
L11 Prism Spectrometer
L13 Geometric Optics
L16 Optical Instruments
W2 Microwave Optics
L12 Interference and Diffraction Patterns

Subjects to be covered

- Geometrical optics: reflection, refraction, mirrors, lenses, aberration, optical instruments
- Oscillations: mechanical (mass-spring, pendulums, energy tossing)
- Oscillations: E&M (*LC* circuits, energy tossing, damped oscillations, forced oscillations)
- Mechanical waves: waves in string, sound waves, water waves, wave reflection, standing waves
- E&M waves: *LC* transmission line, characteristic impedance, wave reflection due to impedance mismatch, radiation of EM waves
- Wave optics: interference, diffraction, resolving power of optical devices

Common questions

- Water waves increase amplitude as they approach a beach. Why?
- What determine the velocity of light? Sound wave?
- How does the police radar measure the speed of a car?
- The frequency of electromagnetic waves in telecommunication has been steadily increasing. Why is high frequency wave more beneficial?
- How much power is the earth receiving from the sun and in what form?
- Lenses of high quality optical instruments are coated with dielectric films. Why?
- What is the fraction of light power reflected at a glass surface?
- Soap films, CD, DVD appear colored. Why? Is it due to the same mechanism as prism and rainbow?
- Why are telescopes for astronomical observation so large?
- Electron microscopes can see better than optical microscopes. Why?
- How can CAT (computer assisted tomography) and NMRI (nuclear magnetic resonance imaging) image internal organs?
- How does the antenna work? What is the basic mechanism of radiation of electromagnetic waves?
- How does a laser work?

Light Wave

- Light velocity in vacuum
- In medium with permittivity ε

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \times 10^8 \text{ m/s}$$

$$c' = \frac{1}{\sqrt{\varepsilon \mu_0}} < c$$

- Harmonic wave *c* = $f\lambda$, *f* (oscillations/sec = Hz) is the frequency and λ (m) is the wavelength. http://physics.usask.ca/~hirose/ep225/
- Index of refraction

$$n = \frac{c}{c'} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} > 1$$

- When light wave enters from air to water (*n* = 1.33), the wavelength is shortened by the factor *n*. The frequency remains unchanged because it is determined by wave sources, e.g., microwave generator, laser, etc.

Efforts to find *c*

- Roemer (Danish astronomer) observed change in the rotation period of the moon Io revolving around Jupiter. Based on Roemer's data, Huygens deduced $c \simeq 2.3 \times 10^8$ m/s
- Bradley (British astronomer) found a fairly accurate estimate of

  $c \simeq 2.8 \times 10^8$ m/s

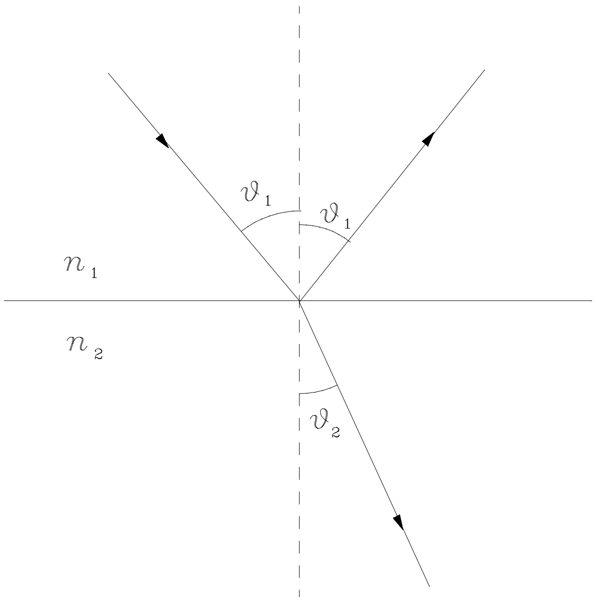
  by aberration effect, shift in the angular location of distant star by $\Delta\theta \simeq v / c$ where v is the earth velocity 3×10^4 m/s.
- Fizeau made the first laboratory measurement of the speed of light using rotating toothed wheel.

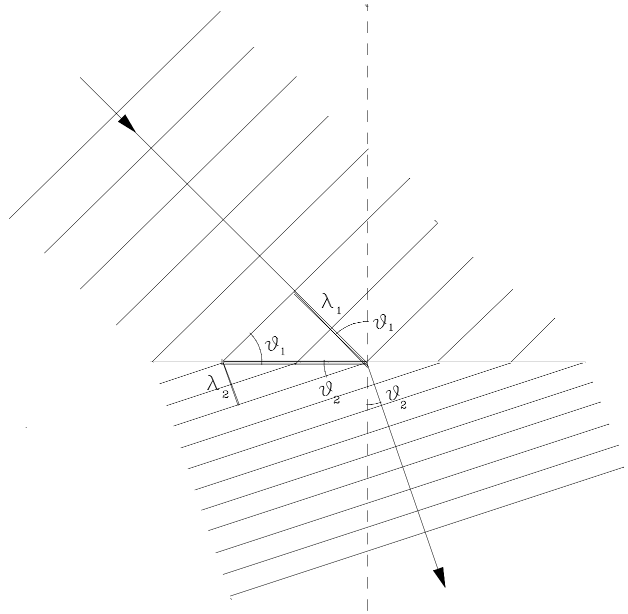
Light reflection and refraction

- Light path obeys the law of minimum transit time.
- Reflection angle = incident angle
- Snell's law for the refraction angle

  In the figure,

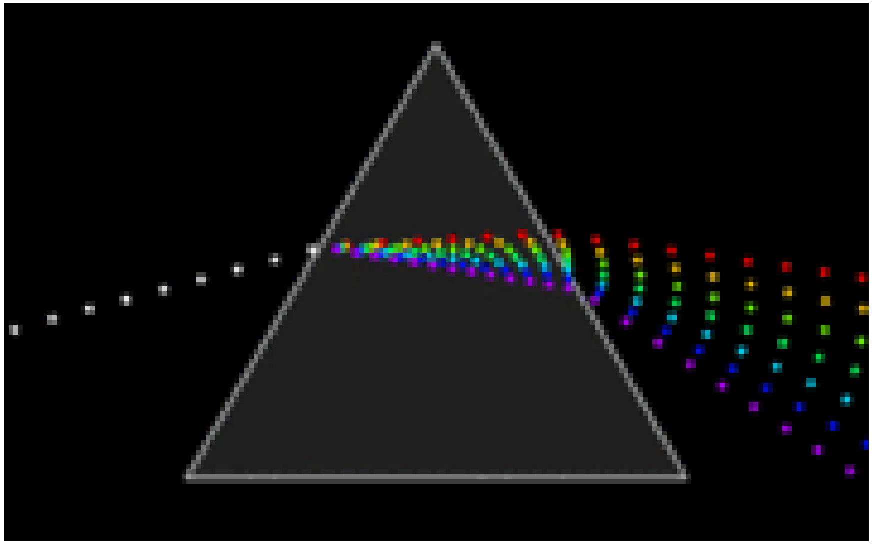
$$\frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2} \rightarrow \frac{c_1}{\sin\theta_1} = \frac{c_2}{\sin\theta_2} \rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2} = \frac{n_2}{n_1}$$

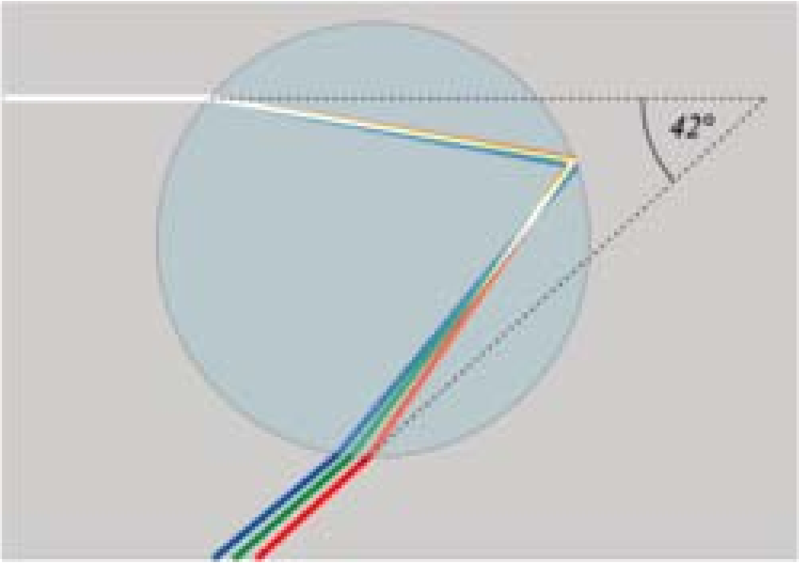




Prism and Rainbow

- Prism and rainbow: splitting of white light into color spectrum. *n* (blue) > *n* (red)
- Colors on CD and DVD are due to something else: diffraction.



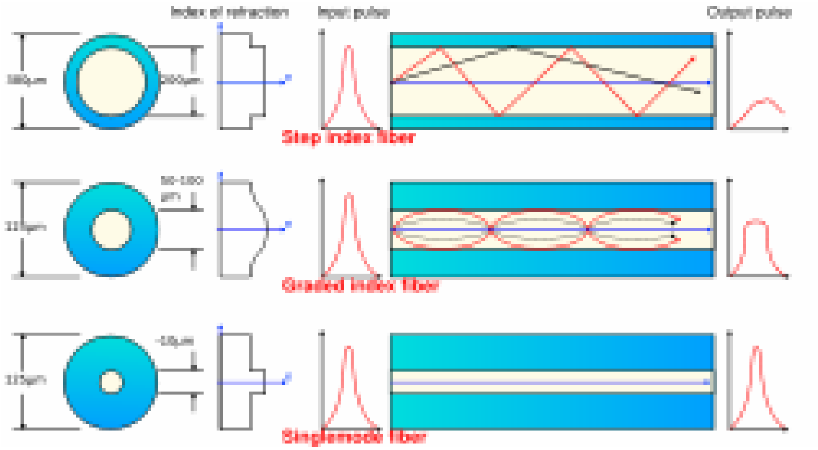


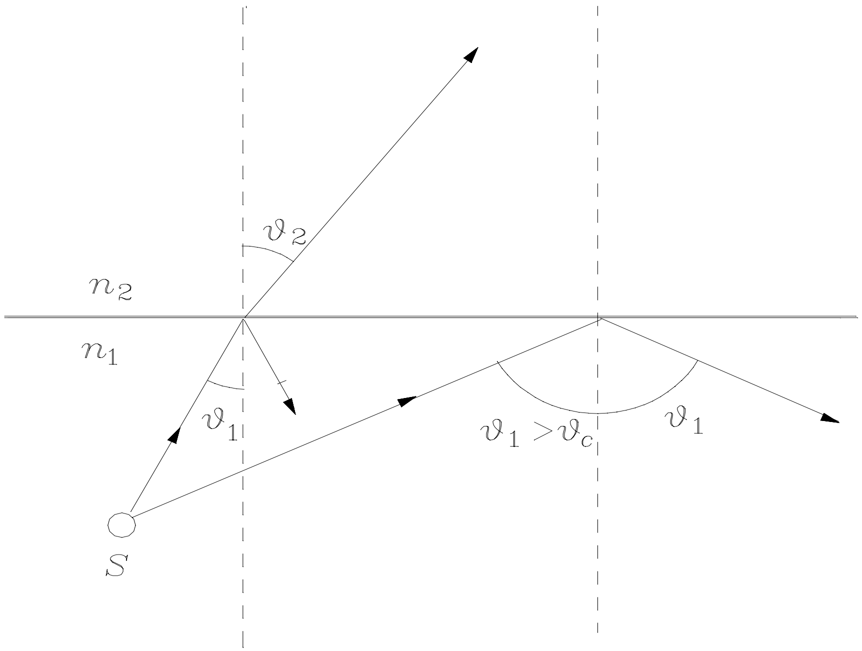
Total reflection

- When light emerges from glass to air, Snell's law becomes

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_g}{n_{air}} = n_g = 1.5$$

- Maximum of θ_2 is 90 degrees. For incident angles > arcsin(1/1.5) = 41.8 deg, light is totally reflected.
- Optical fiber waveguide

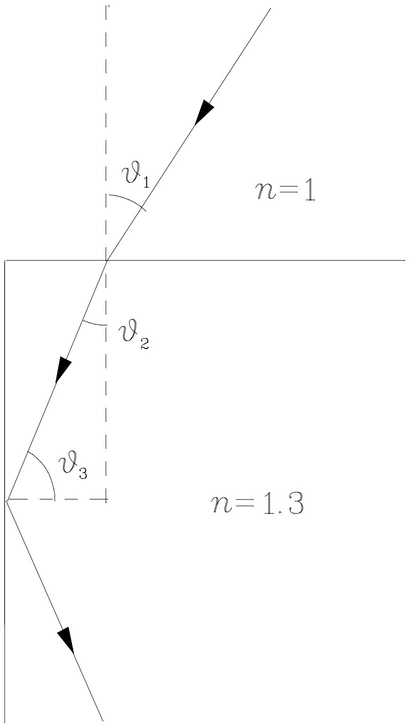




$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

Total reflection

Example: Find the range of incident angle for total reflection at the vertical surface.



$$\theta_3 > \sin^{-1}\left(\frac{1}{1.3}\right) = 50.3^\circ, \theta_2 = 90^\circ - 50.3^\circ = 39.7^\circ$$

$$\theta_1 < \sin^{-1}\left(1.3\sin 39.7^\circ\right) = 56.1^\circ$$

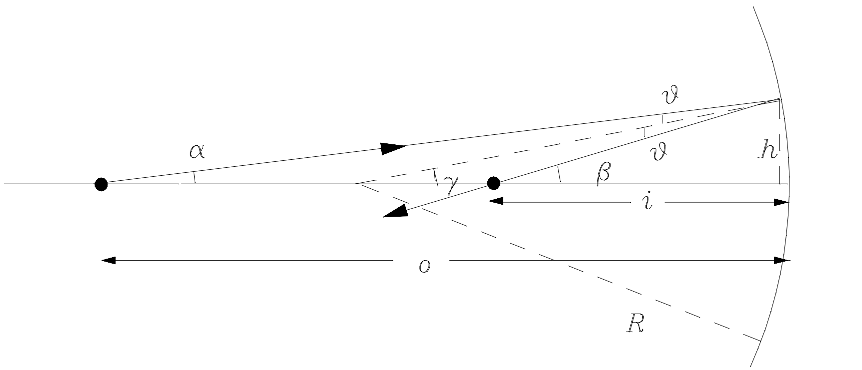
Spherical Mirrors

- Assume small aperture *h* << *R*
- Reflection is symmetric about the radial line
- Place an object at axial position o (not zero) and find the image location *i*.

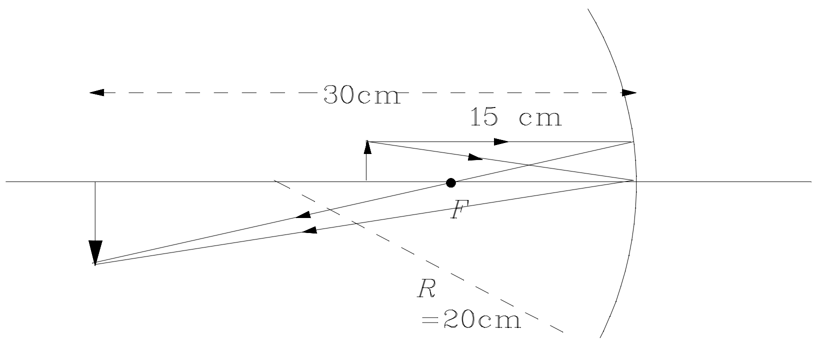
$$\alpha+\theta=\gamma,\theta+\gamma=\beta\rightarrow\alpha+\beta=2\gamma,$$

$$\frac{h}{o}+\frac{h}{i}=\frac{2h}{R}$$

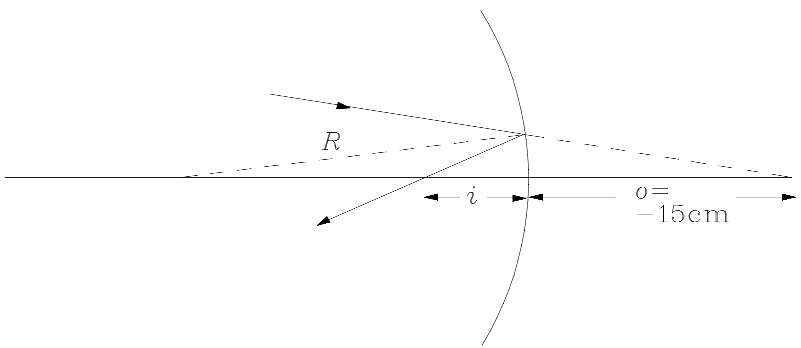
$$\frac{1}{o}+\frac{1}{i}=\frac{2}{R}=\frac{1}{f},\ f=\text{focal length}$$



Example: Concave mirror with *f* = 10 cm. Object at 15 cm, real image at 30 cm. Magnification is *m*=-*i*/*o*=-2 (inverted image).



Example: Virtual object distance (negative *o*)



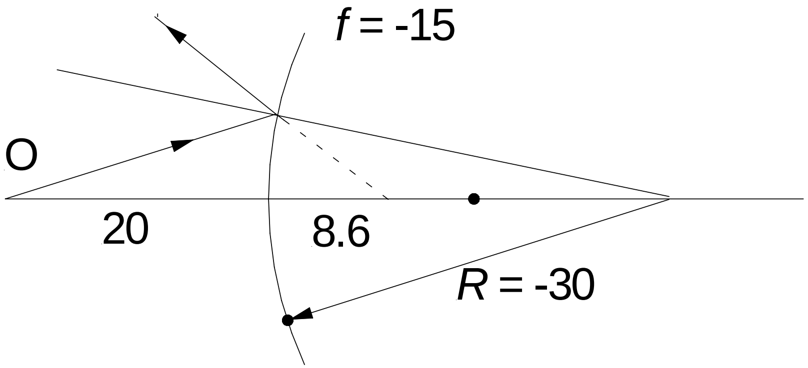
Mirrors (cont)

- Example: Convex mirror (*R*, *f* < 0)

$$\frac{1}{20}+\frac{1}{i}=-\frac{1}{15}$$

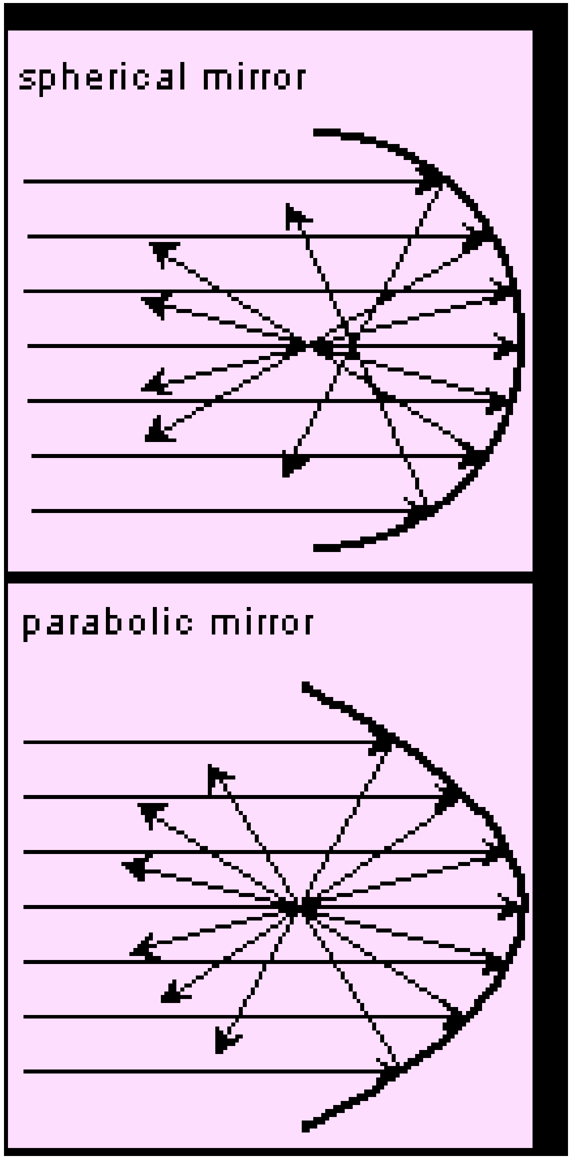
$i = -8.6$ cm = 8.6 cm behind the mirror

$$m=-\frac{-8.6}{20}=+0.43$$



Spherical and Parabolic Mirror

- Spherical mirrors are subject to spherical aberration for large apertures (top).
- For a large object distance (e.g., stars), parabolic mirror is ideal for focusing (bottom).
- Schmidt camera (telescope) corrects spherical aberration by placing a lens corrector.



Sign Convention, Magnification

- Mirror

  $i, o > 0$ real object and real image distance in front of the mirror

  $o, i < 0$ virtual object and image distance behind the mirror

  $R > 0$, $f = R/2 > 0$ concave mirror

  $R < 0$, $f = R/2 < 0$ convex mirror

- Lateral (or angular) magnification

  $m = -i/o$, $m > 0$ erect image, $m < 0$ inverted

Refraction at a Spherical Boundary

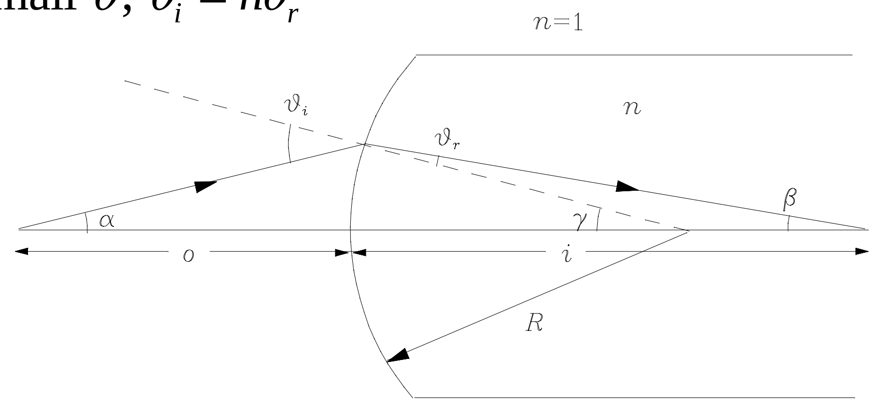
Snell's law $\sin\theta_i = n\sin\theta_r$ For small θ, $\theta_i = n\theta_r$

$$\theta_i = \alpha + \gamma, \theta_r + \beta = \gamma$$

Then

$$\alpha + n\beta = (n-1)\gamma$$

$$\frac{1}{o} + \frac{n}{i} = (n-1)\frac{1}{R}$$



Example: Find the image of

a fish at the center of spherical bowl.

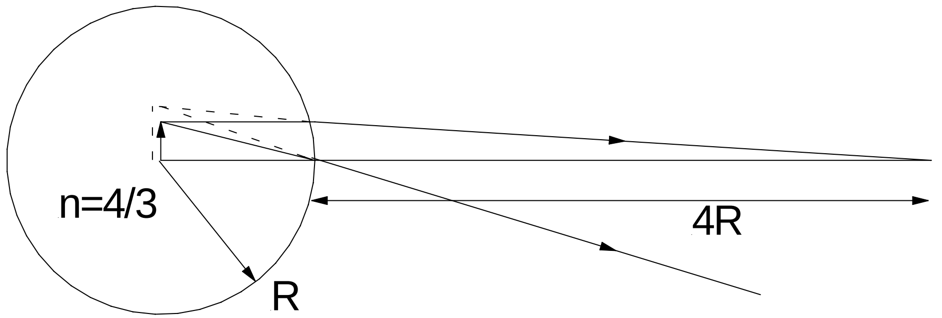
n (water) = 4/3.

From

$$\frac{n}{o} + \frac{1}{i} = (n-1)\frac{1}{R}$$

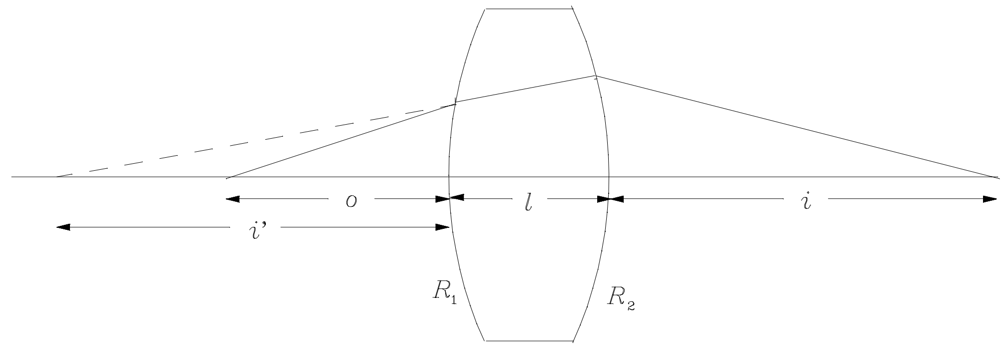
$$\frac{4/3}{R} + \frac{1}{i} = \frac{1}{3}\frac{1}{R}$$

$i = -R$ (at the bowl center)



Magnification is $-\dfrac{i}{o/n} = -\dfrac{-R}{R \times \frac{3}{4}} = +4/3$. Ray tracing confirms the result.

Thin Lens



First refraction

$$\frac{1}{o}+\frac{n}{i'}=(n-1)\frac{1}{R_1}$$

Second refraction

$$\frac{n}{-i'+t}+\frac{1}{i}=(1-n)\frac{1}{R_2}=-(n-1)\frac{1}{R_2}\ \ (i'<0 \text{ in the case shown})$$

If *t* is negligible

$$\frac{1}{o}+\frac{1}{i}=(n-1)\left(\frac{1}{R_2}-\frac{1}{R_2}\right)=\frac{1}{f}.\ \text{Lens maker's formula}$$

If in water,

$$\frac{1}{f}=\frac{n_g-n_w}{n_w}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Sign Convention for Lenses

- For lenses,
  $f > 0$ converging lens, $f < 0$ diverging lens
  $R > 0$ convex surface, $R < 0$ concave
  $o > 0$ real object distance in front of the lens
  $i > 0$ real image distance behind the lens

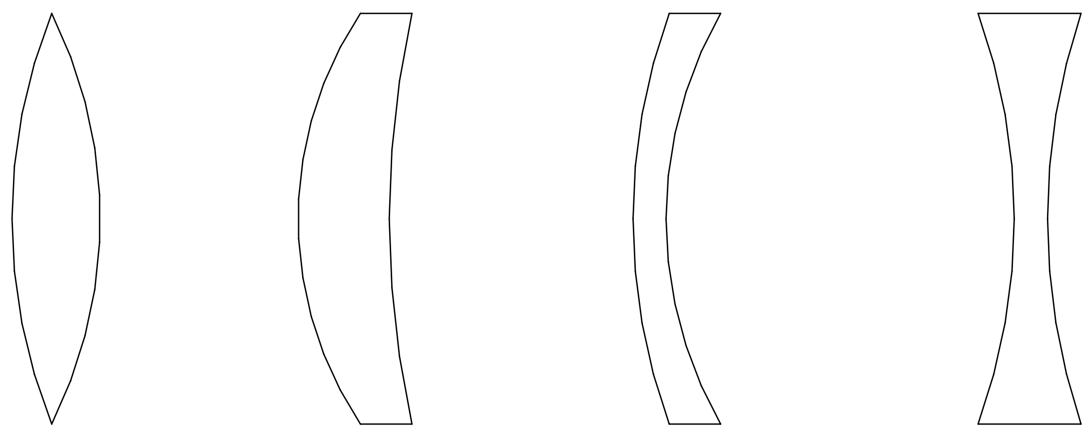
- Magnification

$$m = -\frac{i / n_i}{o / n_o}$$

In the case of the fish in a bowl, $o = R, i = -R, n_o = 4/3, n_i = 1$
and $m = +4/3$.

Thin Lens

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



R1>0
R2<0
f > 0

R1>0
R2>0
R1<|R2|
f > 0

R1>0
R2>0
R1>R2
f < 0

R1<0
R2>0
f < 0

Examples

Example: Design lenses with *f* = +25 cm and – 25 cm given *n* (glass) = 1.5.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{25}$$

$R_1 = 12.5$ cm, $R_2 = \infty$ (plano convex)

$R_1 = -R_2 = 25$ cm (symmetric)

$R_1 = 15$ cm, $R_2 = -75$ cm (meniscus)

$$\frac{1}{f} = 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = -\frac{1}{25}$$

$R_1 = -12.5$ cm, $R_2 = \infty$

$R_1 = 25$ cm, $R_2 = 8.33$ cm, etc.

Example: When a converging lens of *f* = +30 cm is immersed in a liquid, it becomes a diverging lens of *f* = - 130 cm. If *n* (lens glass) = 1.5, what is *n* of the liquid?

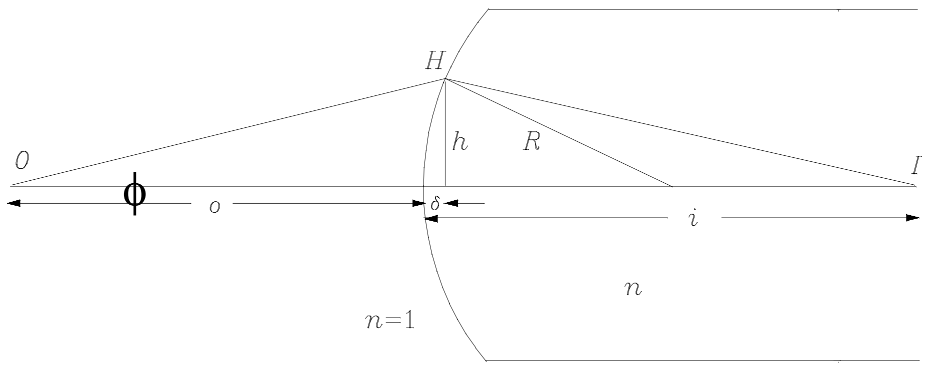
$$\frac{1}{f_{air}} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{30}, \frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{15}$$

In liquid,

$$\frac{1}{f_{liq}} = \frac{n_g - n_{liq}}{n_{liq}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = -\frac{1}{130}$$

Then $n_{liq} = 1.696$.

Physical Meaning of Focusing



Focusing requires the transit time along OHI be independent of the height h or angle ϕ.

Proof:

$$\tau = \frac{OH}{c} + n\frac{HI}{c} = \frac{\sqrt{(o+\delta)^2 + h^2}}{c} + \frac{n}{c}\sqrt{(i-\delta)^2 + h^2}$$

where $\delta = \frac{1}{2}\frac{h^2}{R}$.

$$\sqrt{(o+\delta)^2 + h^2} = o + \frac{h^2}{2R} + \frac{1}{2}\frac{h^2}{o}, \sqrt{(i-\delta)^2 + h^2} = i - \frac{h^2}{2R} + \frac{1}{2}\frac{h^2}{i}$$

$$\tau = \frac{o}{c} + n\frac{i}{c} + \frac{h^2}{2}\left(\frac{1}{o} + \frac{n}{i} - (n-1)\frac{1}{R}\right) = \frac{o}{c} + n\frac{i}{c} \text{ (independent of } h\text{)}$$

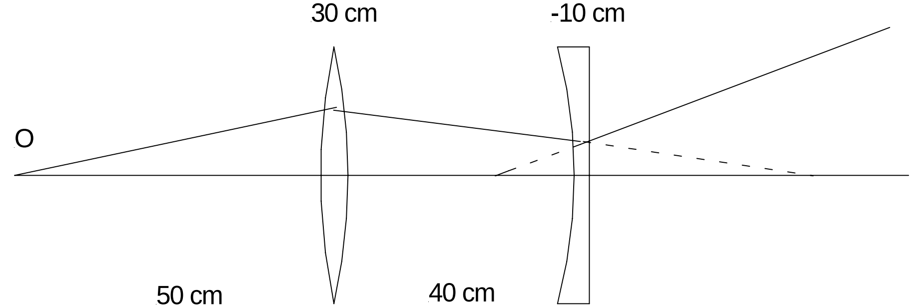
where $\frac{1}{o} + \frac{n}{i} - (n-1)\frac{1}{R} = 0$ is used.

Two lenses, lens-mirror

Example: Two lenses

First lens: $\frac{1}{50}+\frac{1}{i'}=\frac{1}{30}, i'=75\text{ cm}$

Second lens: $-\frac{1}{35}+\frac{1}{i}=-\frac{1}{10}, i=-14\text{ cm}$



Magnification: $m=m_1m_2=\left(-\frac{75}{50}\right)\left(-\frac{-14}{-35}\right)=0.6$ (erect, virtual)

We will revisit this problem in the section of matrix method.

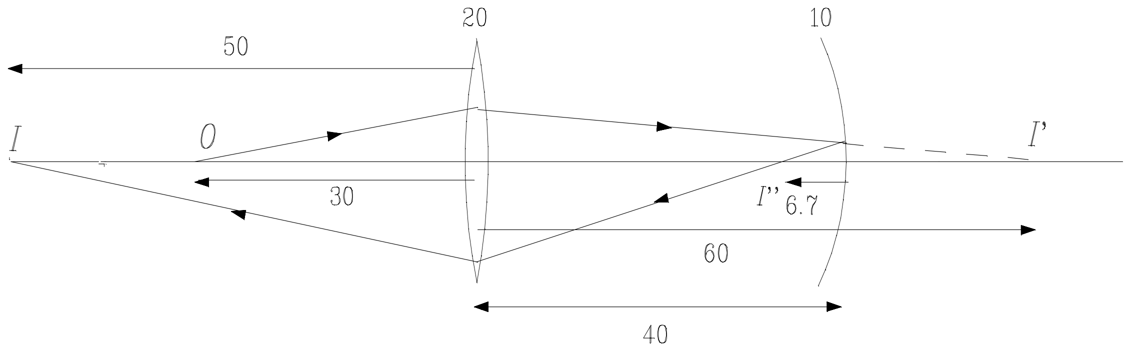
The system matrix is

$$\begin{pmatrix}1 & i\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{1}{10} & 1\end{pmatrix}\begin{pmatrix}1 & 40\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -\frac{1}{30} & 1\end{pmatrix}\begin{pmatrix}1 & 50\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}-0.33-0.067i & 23.33+1.67i\\ -0.067 & 1.667\end{pmatrix}$$

From $B=23.33+1.67i=0$ (the condition for focusing), $i=-14$ cm.

Lens-Mirror System



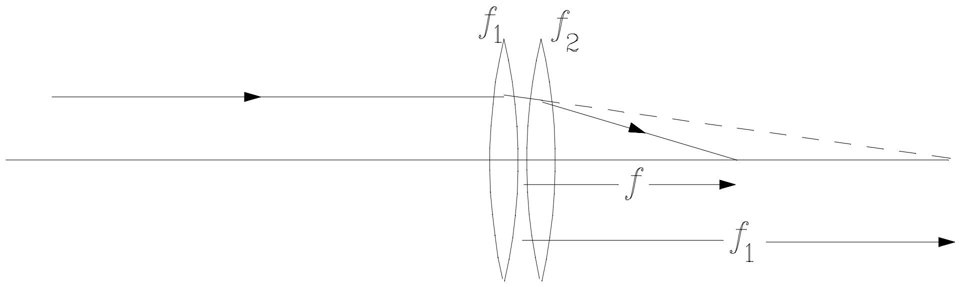
Lens $\frac{1}{30}+\frac{1}{i}=\frac{1}{20}, i=60.0$

Mirror $\frac{1}{-20}+\frac{1}{i'}=\frac{1}{10}, i'=6.67$ (-20 means a virtual object)

Lens again $\frac{1}{33.33}+\frac{1}{i''}=\frac{1}{20}, i''=50.0$ (final image)

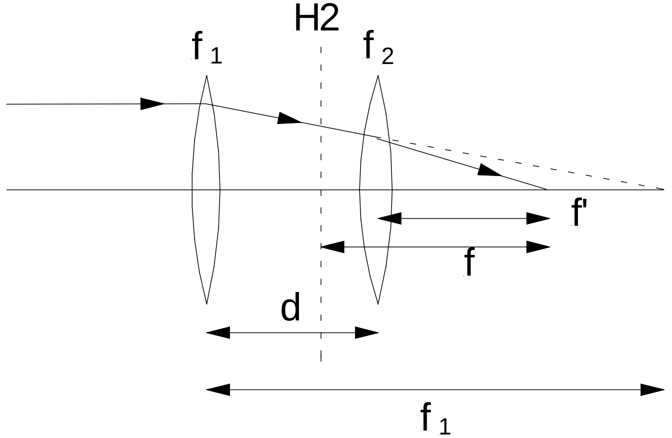
Magnification $m=(-2)(+0.333)\left(-\frac{50}{33.33}\right)=+1.0$ (erect, real)

Compound lens



- Two lenses touching:
  Effective focal length

$$-\frac{1}{f_1}+\frac{1}{i}=\frac{1}{f_2}, \quad \frac{1}{f_{eff}}=\frac{1}{f_1}+\frac{1}{f_2}$$



- Two lenses separated by *d*

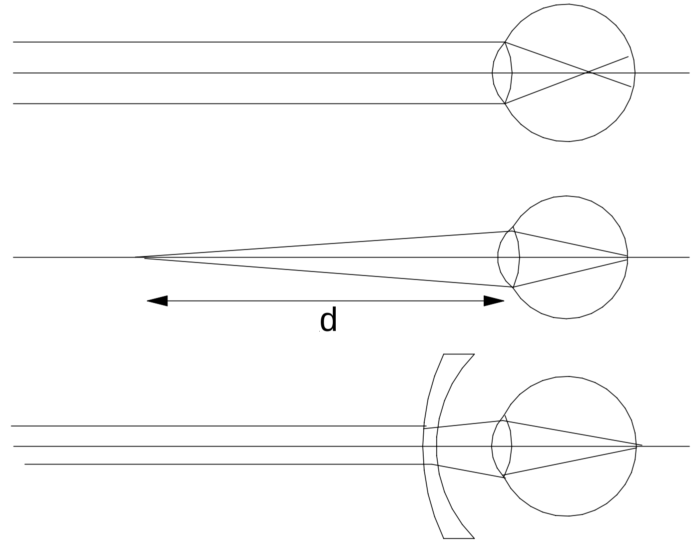
$$\frac{1}{f_1-d}+\frac{1}{f'}=\frac{1}{f_2}, \quad \frac{1}{f'}=\frac{1}{f_2}-\frac{1}{f_1-d}$$

f' is NOT the focal length of the compound lens. The focal length is (to be shown in matrix section)

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}, \; f-f'=\frac{fd}{f_1} \text{ principal plane}$$

Myopia

Myopia correction

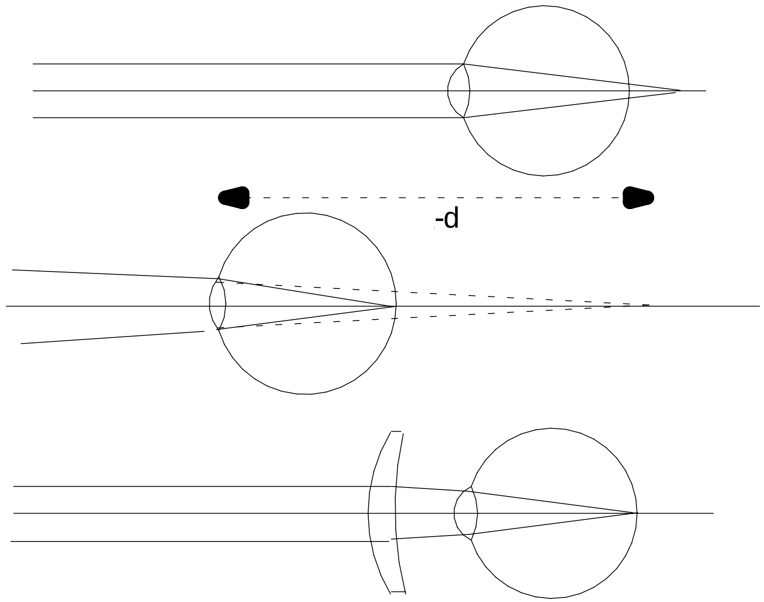


Myopia is caused by too short a focal length of the eye lens. Placing a diverging lens can correct it. If the far point of the eye is d, the focal length of the correcting lens is $f = -d$. Let the eye lens focal length be f_e and the eye lens retina distance be o_e. Without lens,

$\frac{1}{d} + \frac{1}{o_e} = \frac{1}{f_e}$. With lens $\frac{1}{\infty} + \frac{1}{o_e} = \frac{1}{f_e} + \frac{1}{f}$. Then $f = -d$.

Hyperopia

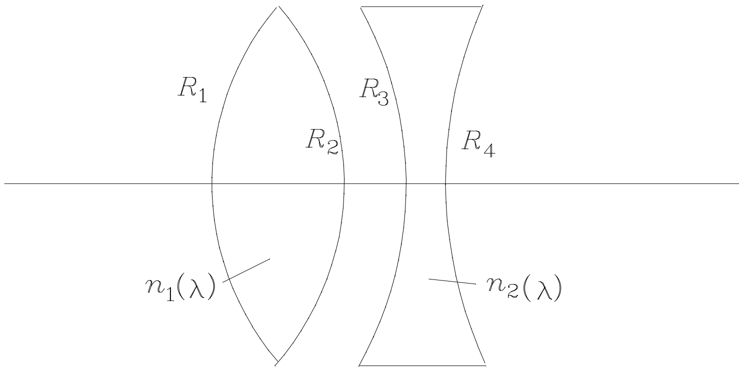
Hyperopia correction



Hyperopia is caused by too long a focal length of the eye lens. Placing a converging lens can correct it. If the near point of the eye is $-d$, the focal length of the correcting lens should be $f = d$.

Achromatic Doublet

Chromatic aberration of lens can be corrected by combining converging and diverging lenses made of different glasses as shown. The focal length is



Red: $$\frac{1}{f_R} = (n_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_R' - 1)\left(\frac{1}{R_3} - \frac{1}{R_4}\right)$$

Blue: $$\frac{1}{f_B} = (n_B - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_B' - 1)\left(\frac{1}{R_3} - \frac{1}{R_4}\right)$$

From $f_R = f_B$,

$$(n_B - n_R)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n_R' - n_B')\left(\frac{1}{R_3} - \frac{1}{R_4}\right)$$

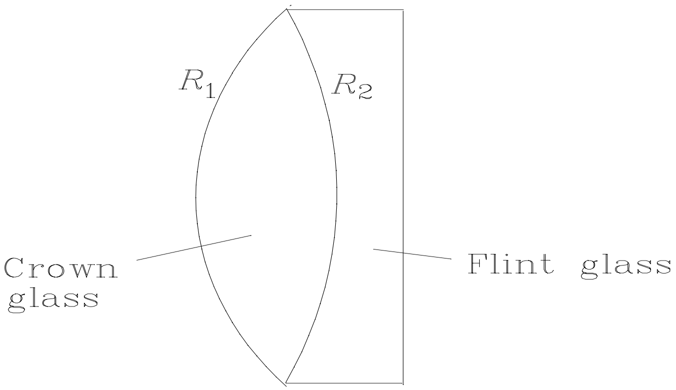
Example: Converging lens made of Crown glass ($n_R = 1.505$, $n_B = 1.510$) and diverging lens made of Flint glass ($n_R = 1.615, n_B = 1.630$). If focal length 50 mm is needed, determine the radii R_1, R_2.

Achromatic condition

$$0.005\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = -0.015\left(\frac{1}{R_2}-\frac{1}{\infty}\right), R_2 = -2R_1$$

Focal length of blue light (= red light)

$$\frac{1}{50} = 0.51\left(\frac{1}{R_1}-\frac{1}{R_2}\right) + 0.63\frac{1}{R_2},\ \text{Then } R_1 = 22.5 \text{ mm, } R_2 = 50 \text{ mm}$$



Achromatic Compound Lens

The effective focal length of compound lens is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

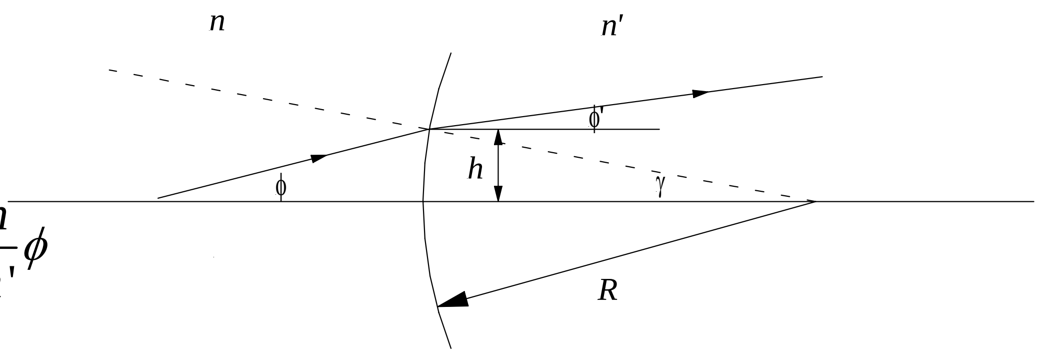
$$= (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4}\right) - (n-1)^2 d \left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(\frac{1}{R_3} - \frac{1}{R_4}\right)$$

If $d = \frac{f_1 + f_2}{2}$, the focal length f becomes independent of the wavelength.

(Assignment Problem)

Matrix Method

- Refraction



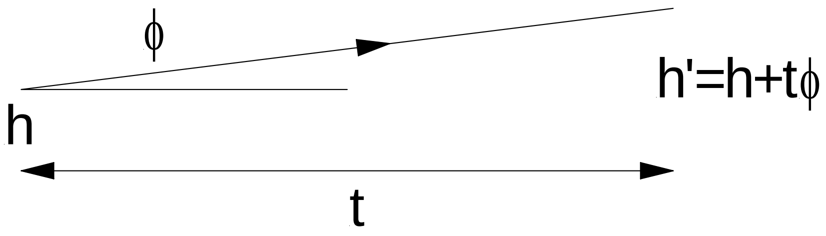
$$h = h'$$

$$n(\phi + \gamma) = n'(\phi' + \gamma) \quad \phi' = \left(\frac{n}{n'} - 1\right)\frac{h}{R} + \frac{n}{n'}\phi$$

In matrix form

$$\begin{pmatrix} h' \\ \phi' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \left(\frac{n}{n'} - 1\right)\frac{h}{R} & \frac{n}{n'} \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix} = \mathbf{R} \begin{pmatrix} h \\ \phi \end{pmatrix}$$

- Transmission over distance *t*



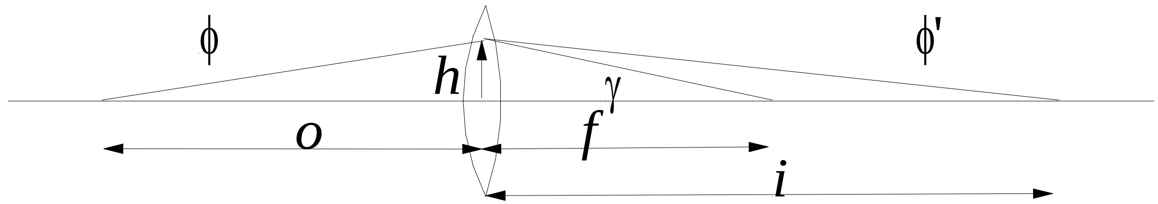
$$\begin{pmatrix} h' \\ \phi' \end{pmatrix} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix} = \mathbf{T} \begin{pmatrix} h \\ \phi \end{pmatrix}$$

Matrix of Thin Lens

$$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}\rightarrow \phi-\phi'=\frac{1}{f}h$$

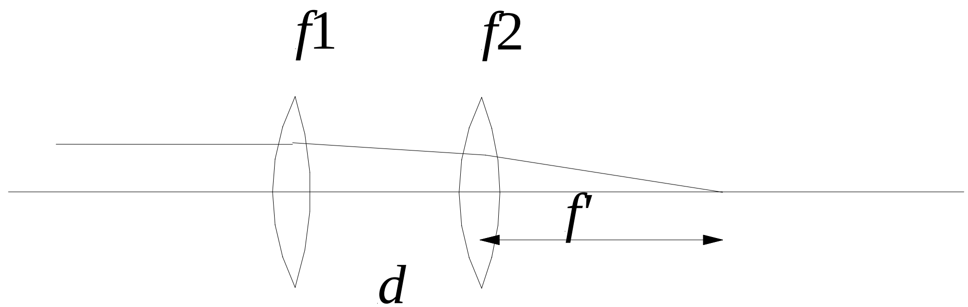
$$\phi'=-\frac{1}{f}h+\phi \text{ and } h'=h$$

$$\begin{pmatrix} h' \\ \phi' \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}\begin{pmatrix} h \\ \phi \end{pmatrix}$$



Two lenses separated by d

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$



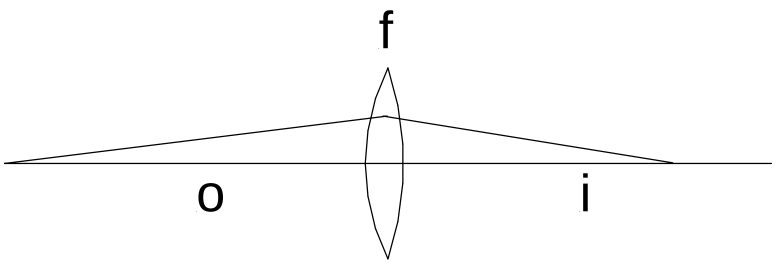
$$=\begin{pmatrix} 1-\frac{d}{f_1} & d \\ -\frac{1}{f_1}-\frac{1}{f_2}+\frac{d}{f_1 f_2} & 1-\frac{d}{f_2} \end{pmatrix}, \quad \frac{1}{f_{eff}}=-C=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2},\ f'=-\frac{A}{C}$$

Single Thin Lens

If an object is at o from the first lens and image at i from the second lens, total matrix is

$$\begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & o \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{i}{f} & o+i-\frac{oi}{f} \\ -\frac{1}{f} & 1-\frac{o}{f} \end{pmatrix} = \begin{pmatrix} -\frac{i}{o} & 0 \\ -\frac{1}{f} & -\frac{o}{i} \end{pmatrix}$$

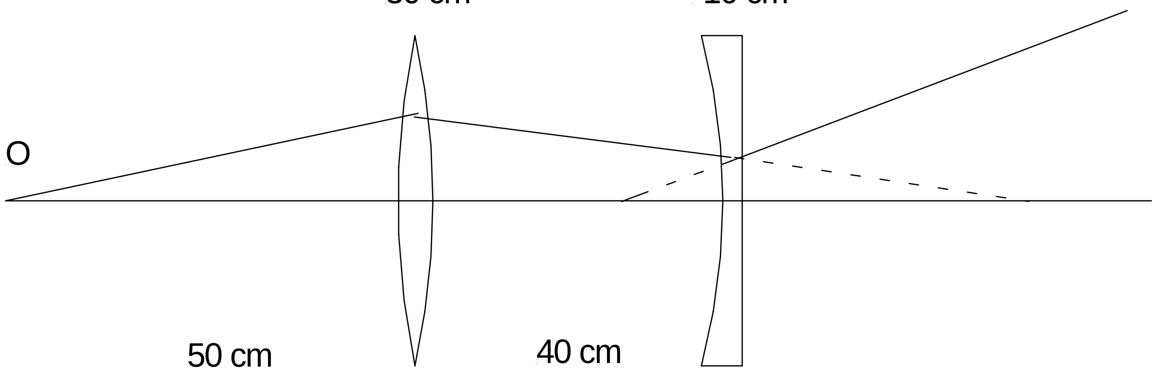


$B=0$ determines the image distance

$\frac{1}{i}=\frac{1}{f}-\frac{1}{o}$, lens formula

Magnification = $A = 1-\frac{i}{f} = -\frac{i}{o}$

Revisit



The system matrix is

$$\begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1}{10} & 1 \end{pmatrix}\begin{pmatrix} 1 & 40 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{30} & 1 \end{pmatrix}\begin{pmatrix} 1 & 50 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -0.33-0.067i & 23.33+1.67i \\ -0.067 & 1.667 \end{pmatrix}$$

From $B = 0 \rightarrow i = -14$ cm

$A = 0.6$ (magnification)

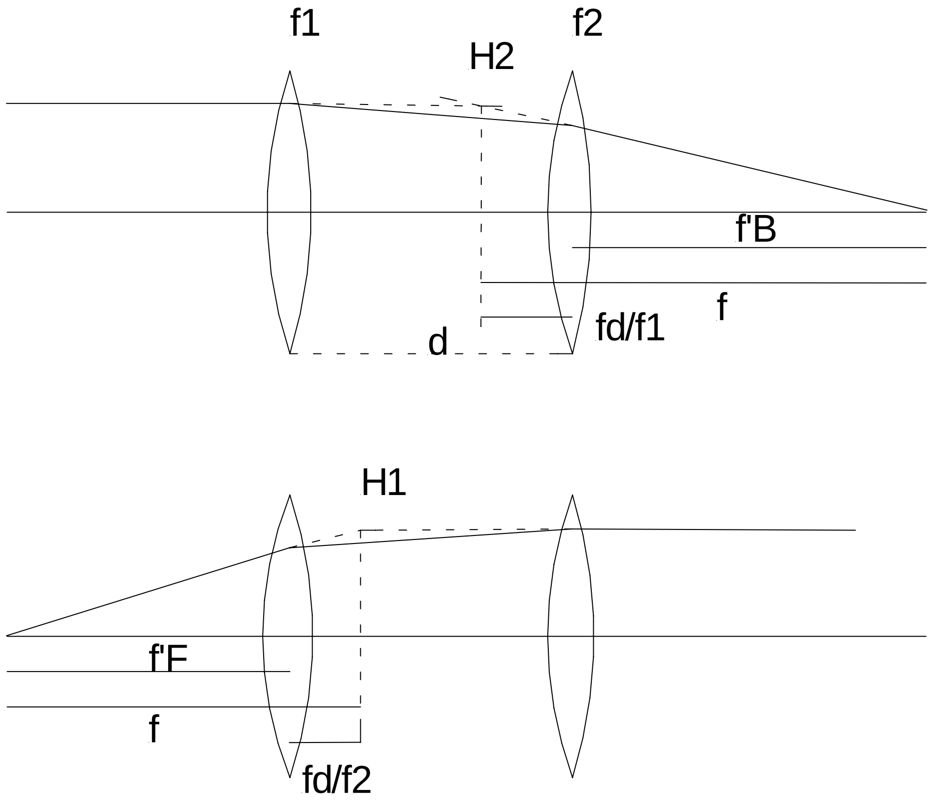
Focal length = $-1/C = 15$cm

Principal Planes (thin compound lens)

f'B: back focal position

f'F: front focal position

f: effective focal length



If the object distance is relative to H1 and image distance relative to H2, the formal lens formula still holds

$$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$$

For the compound lens in the previous example,

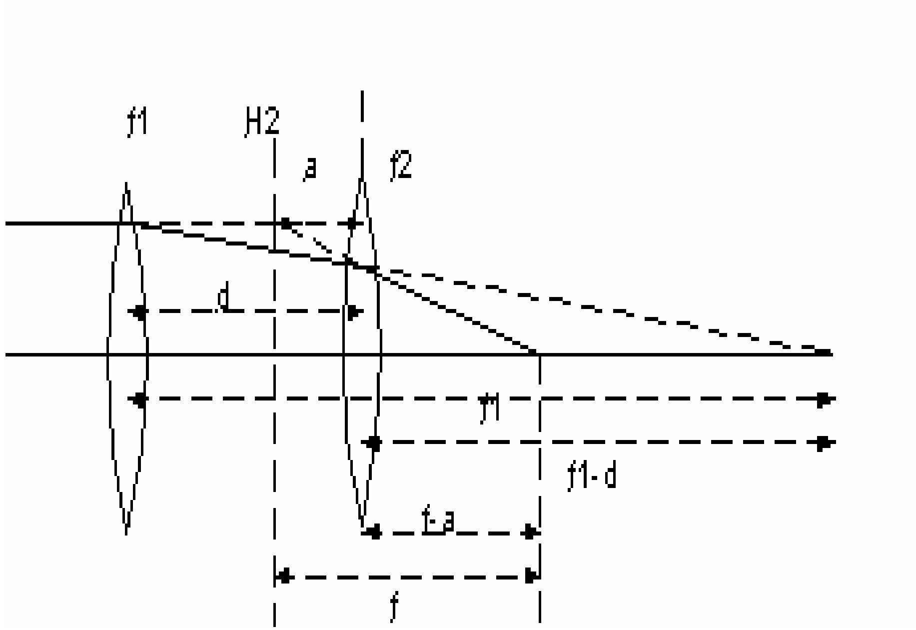
H2 at $\dfrac{fd}{f_1} = \dfrac{15 \times 40}{30} = 20$ cm to the left of lens 2

H1 at $\dfrac{fd}{f_2} = \dfrac{15 \times 40}{-10} = -60$ cm to the right of lens 1 = 60 cm to the left of lens 1

Then the object distance realtive to H1 is -10 cm.

From $\dfrac{1}{-10} + \dfrac{1}{i} = \dfrac{1}{15}$, we find $i = 6$ cm to the right of H2 or14 cm to the left of lens 2.

How to Locate H2 Plane

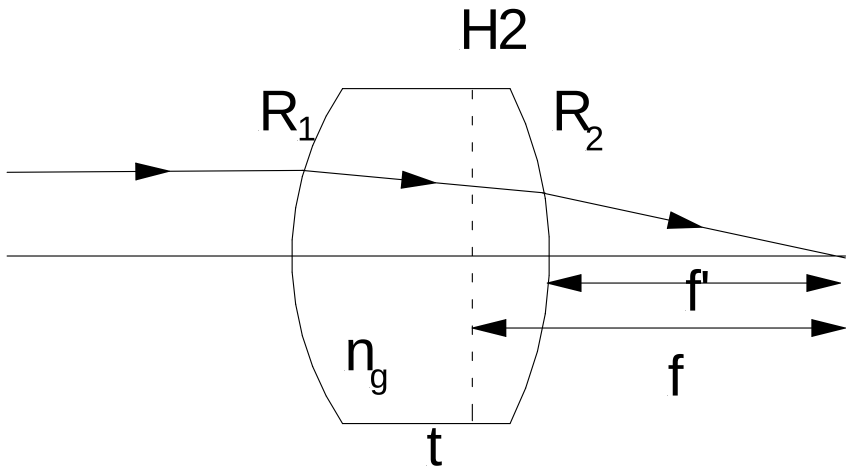


$$a = \frac{fd}{f_1} \rightarrow \frac{f-a}{a} = \frac{f}{\frac{fd}{f_1}} - 1 = \frac{f_1 - d}{d}$$

Therefore, the H plane can be found from the intersection as shown.

Thick Lens

Total lens matrix is



$$\begin{pmatrix} 1 & 0 \\ (n_g-1)\frac{1}{R_2} & n_g \end{pmatrix}\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \left(\frac{1}{n_g}-1\right)\frac{1}{R_1} & \frac{1}{n_g} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{t}{R_1}\left(1-\frac{1}{n_g}\right) & \frac{t}{n_g} \\ -(n_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)-\frac{(n_g-1)^2 t}{n_g R_1 R_2} & 1+\frac{t}{R_2 n_g}(n_g-1) \end{pmatrix}$$

The effective focal length is

$$\frac{1}{f}=(n_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{(n_g-1)^2 t}{n_g R_1 R_2}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{t}{n_g f_1 f_2}, \text{ cf. } \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \text{ (compound lens)}$$

H2 at $\frac{ft}{f_1 n}$ to the left of second vertex. H1 at $\frac{ft}{f_2 n_g}$ to the right of first vertex.

Example: A lens 3 cm thick has R1=10 cm, R2=-5 cm, n (glass) = 1.5. Determine its focal length and location of the principal planes for the two surfaces.

The lens matrix is

$$\begin{bmatrix} 1 & 0 \\ (n-1)\frac{1}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \left(\frac{1}{n}-1\right)\frac{1}{R_1} & \frac{1}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ (1.5-1)\frac{1}{-5} & 1.5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \left(\frac{1}{1.5}-1\right)\frac{1}{10} & \frac{1}{1.5} \end{bmatrix} = \begin{bmatrix} 0.900 & 2.00\text{cm} \\ -0.140\text{cm}^{-1} & 0.800 \end{bmatrix}$$

The focal length is $f = -\frac{1}{C} = -\frac{1}{-0.140} = 7.143$ cm.

Paraxial ray from left is focused at $f' = -\frac{A}{C} = 6.429$ cm.

The principal plane H2 is at 7.143-6.429=0.714 cm = 7.14 mm from the vertex to the left

Similarly, H1 is at 1.43 cm to the right of the first vertex.

For light coming from right, the order of matrices is reversed.

$$\begin{bmatrix} 1 & 0 \\ (1.5-1)\dfrac{1}{-10} & 1.5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \left(\dfrac{1}{1.5}-1\right)\dfrac{1}{5} & \dfrac{1}{1.5} \end{bmatrix}$$

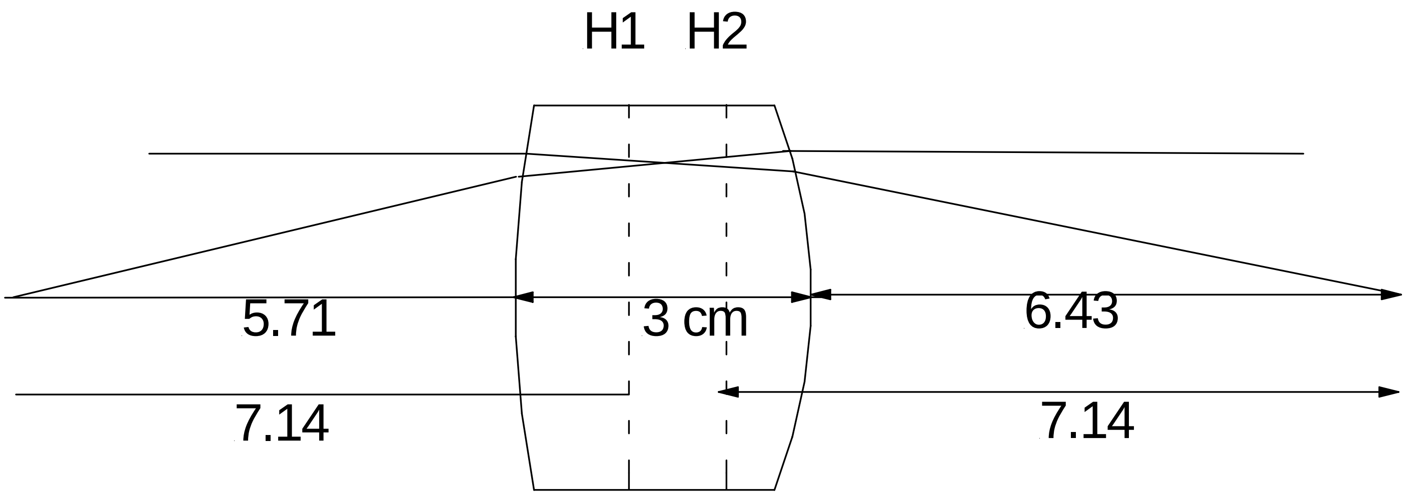
$$=\begin{bmatrix} 0.8 & 2.00\text{cm} \\ -0.140\text{cm}^{-1} & 0.900 \end{bmatrix}$$

The focal length is $f = -\dfrac{1}{C} = -\dfrac{1}{-0.140} = 7.143$ cm (unchanged)

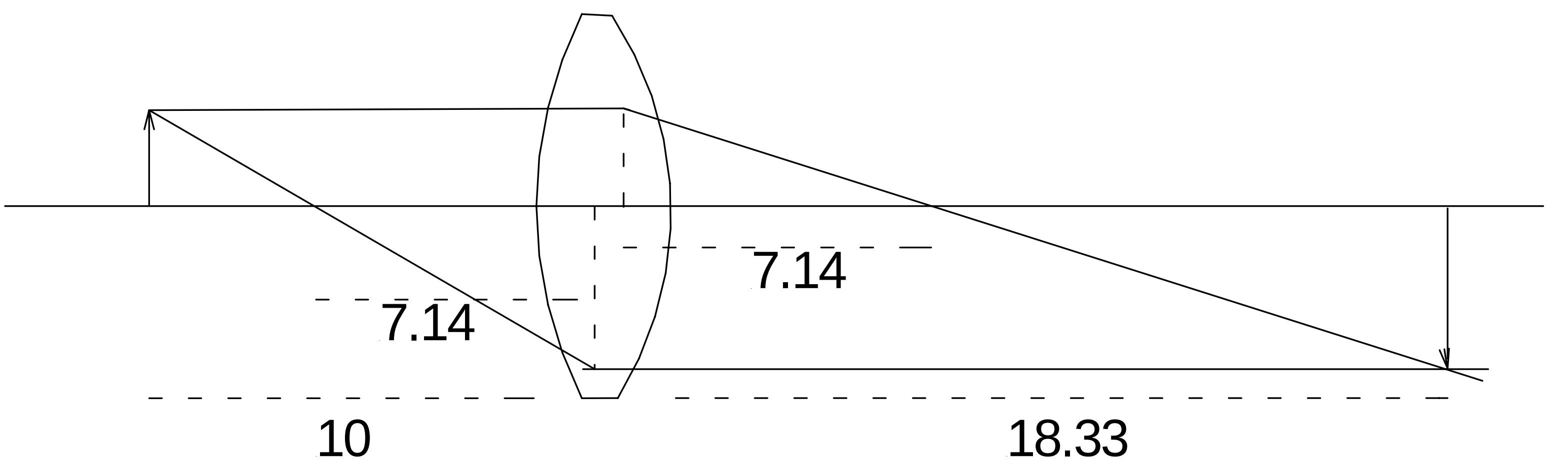
Paraxial ray from right is focused at $f' = -\dfrac{A}{C} = 5.714$ cm.

The principal plane H1 is at 7.143-5.714=0.714 cm = 1.429 cm from the vertex to the right

Thick Lens Paraxial Ray Diagram



If object at 10 cm from the first vertex, the image is at 18.33 cm from the second vertex.



Example: An object is placed at 20 cm in front of the thick lens of the previous example. Determine the final image location.

In the lens formula,

$$\frac{1}{o}+\frac{1}{i}=\frac{1}{f_{eff}},\ o \text{ is to be measured from H1 and } i \text{ from H2.}$$

$$o = 20 + 1.43 = 21.43 \text{ cm}$$

From $\frac{1}{21.43}+\frac{1}{i}=\frac{1}{7.143} \rightarrow i = 10.71$ cm.

Magnification is $m = -10.71 / 21.43 = -0.5$

In matrix method,

$$\begin{bmatrix} 1 & i' \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.900 & 2.00 \\ -0.140 & 0.800 \end{bmatrix}\begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9 - 0.14i' & 20\text{-}2i' \\ -0.140 & -2 \end{bmatrix}$$

From $B = 0, \rightarrow i' = 10$ measured from the second vertex.

Magnification is $A = 0.9 - 1.4 = -0.5$.

Magnifying Glass

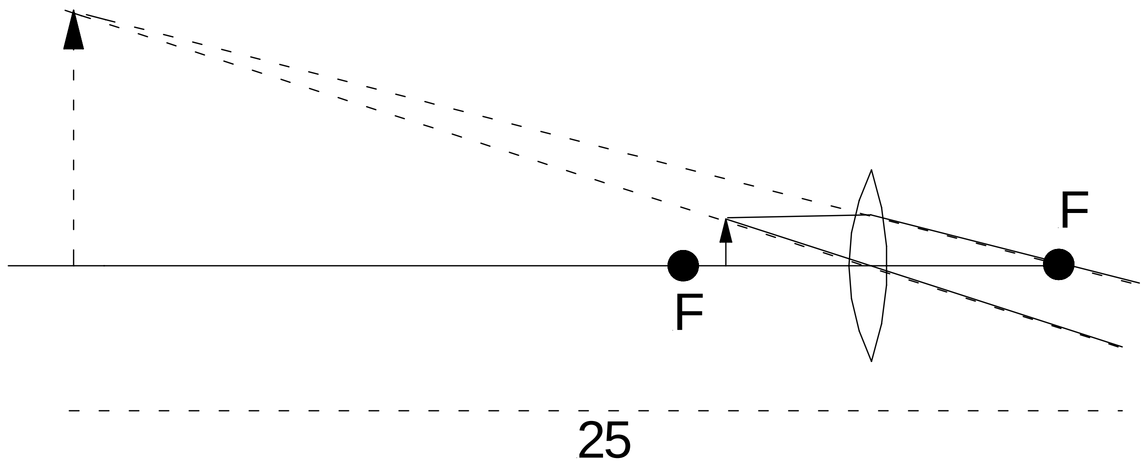
If an object is placed slightly inside the focal point, an erect virtual image is formed.

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

$$i = \frac{of}{o - f} < 0 \text{ if } o < f$$

The average image distance is -25 cm.

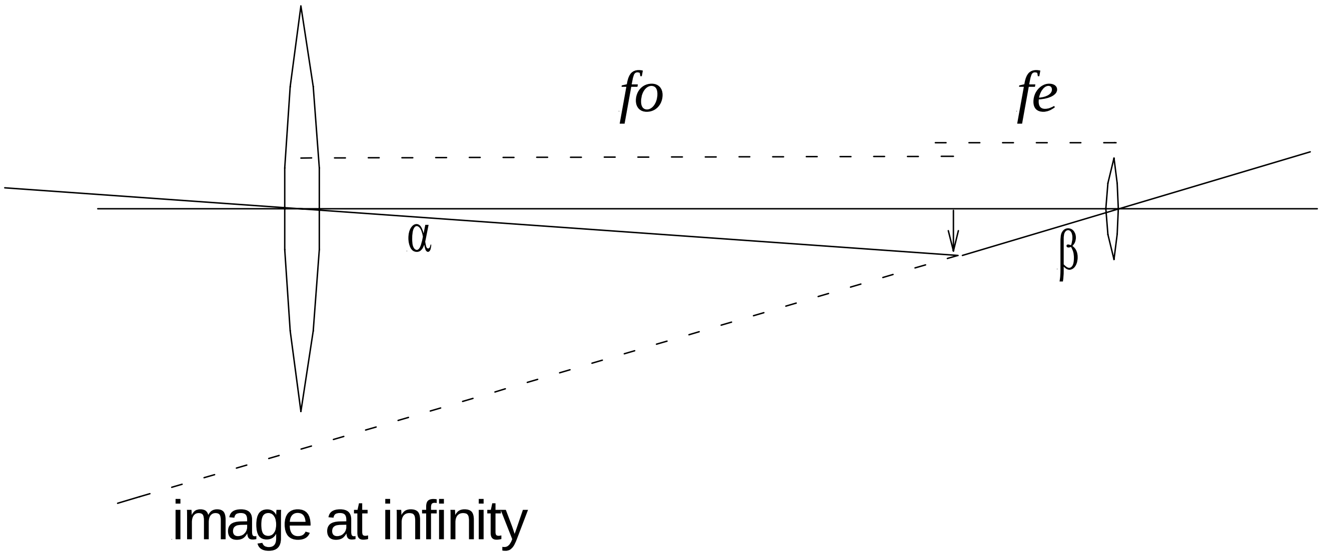
Magnification is $m = \frac{25}{f} > 0.$



Telescope

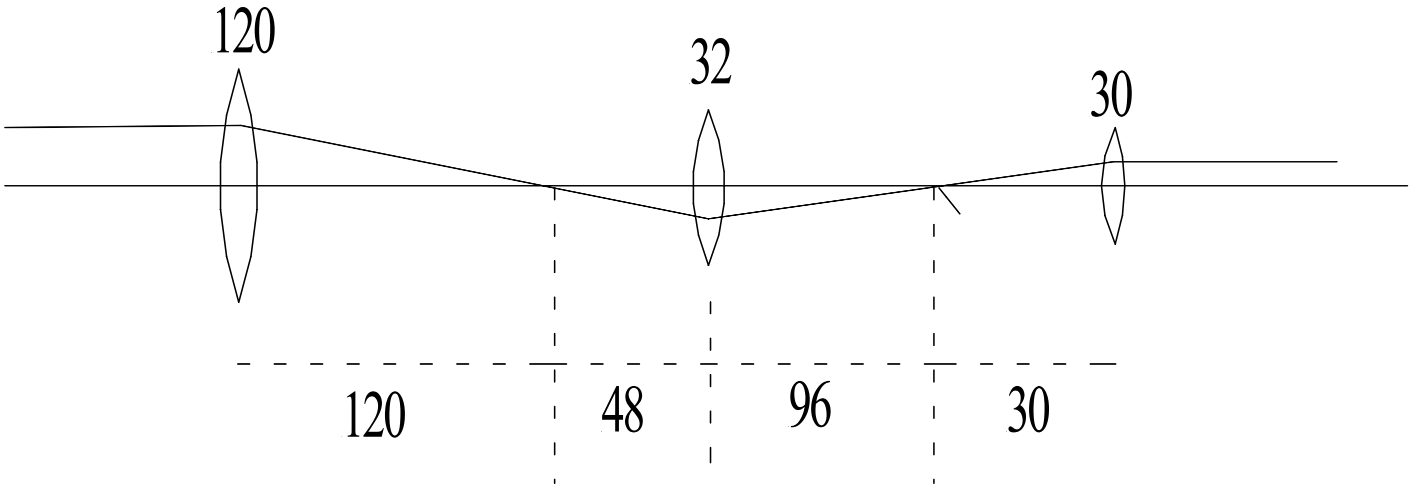
In a telescope, the objective lens forms image of distant object at its focal position. The function of the eyepiece is to magnify it as in magnifying glass except the image is at infinity. The image by the objective lens should be formed near the focal point of the eyepiece. The magnification is

$$m = -\frac{\beta}{\alpha} = \left(-\frac{f_o}{\infty}\right)\left(-\frac{-\infty}{f_e}\right) = -\frac{f_o}{f_e}$$



Telescope with Erector (for Rifle)

To get erect image as needed in rifle telescope, place another lens in between. The total telescope length of ~30 cm and magnification of 8 or 10 is reasonable. An example is shown. In this example, the erector has magnification of -2 and the total magnification is +8. (homework)



Microscope

In a microscope, an object is placed slightly beyond the focal length of the objective lens to form a magnified real intermediate image at the end of the microscope tube of length *L*. The function of the eyepiece is to magnify it as in magnifying glass. Magnification is

$$m = -\frac{L}{f_o}\frac{25}{f_e} \text{ all in cm}$$