Problems in Chapter 3 (Fluid Dynamics)

1. An incompressible, inviscid fluid flows steadily with circular streamlines around a horizontal bend as shown. The radial variation of the velocity profile is given by \(rv = r_o v_o\) where \(v_o\) is the velocity at the inside of the bend which has a radius \(r = r_o\). Determine the pressure variation across the bend in terms of \(v_o\), \(r_o\), \(\rho\), \(r\) and \(p_o\), where \(p_o\) is the pressure at \(r = r_o\). Neglect gravity.

Sol) 1. Related with the Bernoulli e.q. across the streamline
   2. Choose two points (1) (at \(r = r_o\)) & (2) (at \(r = r\)) across the streamline
   3. Apply the Bernoulli equation across the streamline (i.e. between (1) & (2))

\[
p_o + \rho v_o^2 \int_{r_o}^{r} \frac{z}{r} \, dn + \gamma z_1 (at \ r = r_o) = p + \rho v^2 \int_{r_o}^{r} \frac{z}{r} \, dn + \gamma z_2 (at \ r = r)
\]

where \(dn = -dr\), \(z_1 = 0\), \(z_2 = 0\) (Horizontal bend)

4. Using the velocity profile, \(v = (r_o v_o) / r\)

then, \(p_o = p - \rho (r_o v_o)^2 \int_{r_o}^{r} \frac{1}{r^3} \, dr = p + \frac{1}{2} \rho (r_o v_o)^2 \frac{1}{r^2} - \frac{1}{r_o^2}\)

Finally, \(p = p_o - \frac{1}{2} \rho (r_o v_o)^2 \frac{1}{r^2} - \frac{1}{r_o^2}\) or \(p = p_o + \frac{1}{2} \rho v_o^2 \left[1 - \left(\frac{r_o}{r}\right)^2\right]\) [ANSWER]
2. Water flows from the faucet on the first floor of the building shown with a maximum velocity of 20 ft/s. For steady inviscid flow, determine the maximum water velocity from the basement faucet and from the faucet on the second floor (assume each floor is 12 ft tall).

**Sol:** From the simplified diagram as shown,

Bernoulli’s equation along the streamline;

\[
p_1 + \frac{1}{2} \rho v_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho v_2^2 + \gamma z_2 \quad [\text{Between (1) and (2) (Basement)}]
\]

\[
= p_3 + \frac{1}{2} \rho v_3^2 + \gamma z_3 \quad [\text{Between (1) and (3) (1st floor)}]
\]

\[
= p_4 + \frac{1}{2} \rho v_4^2 + \gamma z_4 \quad [\text{Between (1) and (4) (2nd floor)}]
\]

Since all pressures at the faucets \(p_2, p_3, p_4\) are zero (Free jet)

a) Water velocity at the basement faucet

\[
v_2 = \sqrt{\frac{2}{\rho} \left[ p_3 + \frac{1}{2} \rho v_3^2 + \gamma(z_3 - z_2) - p_2 \right]} = \sqrt{v_3^2 + 2g(z_3 - z_2)} \quad [\text{since } p_2 = p_3 = 0]
\]

(where \(v_3 = 20 \text{ ft/s}, \text{ and } z_3 - z_2 = 12 \text{ ft}\))

\[
\therefore \quad v_2 = \sqrt{(20)^2 + 2(32.2)(12)} = 34.2 \text{ ft/s} \quad [\text{ANSWER}]
\]

b) Water velocity at the faucet on the second floor

\[
v_4 = \sqrt{\frac{2}{\rho} \left[ p_3 + \frac{1}{2} \rho v_3^2 + \gamma(z_3 - z_4) - p_4 \right]} = \sqrt{v_3^2 + 2g(z_3 - z_4)} \quad [\text{since } p_4 = p_3 = 0]
\]

(where \(v_3 = 20 \text{ ft/s}, \text{ and } z_3 - z_4 = -12 \text{ ft}\))

\[
= \sqrt{(20)^2 + 2(32.2)(-12)} = \sqrt{-372.8} \quad \text{Imaginary } \rightarrow \text{Unreal physical value}
\]

\[
\therefore \quad \text{No water at the second floor} \quad [\text{ANSWER}]
\]
3. (Bernoulli eq. & Continuity) Water flows into a large tank at a rate of 0.011 m³/s as shown. The water leaves the tank through 20 holes in the bottom of the tank, each of which produces a stream of 10-mm diameter. **Determine the equilibrium height, h, for steady state operation.**

Sol)  
1. The equilibrium height \( h \) is related with the continuity eq.

\[ Q_1 \text{ (into a tank)} = Q_2 \text{ (leaving through holes)} \]

or

\[ 0.011 = 20 \text{ (holes)} \cdot A_2 \sqrt{v_2} = 20 \frac{\pi}{4} D_2^2 v_2 \]

2. In order to determine \( v_2 \), the Bernoulli eq. along a streamline

\[ p_1 + \frac{1}{2} \rho v_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho v_2^2 + \gamma z_2 \]

where \( p_1 = p_2 = 0 \) (atmosphere), \( z_1 = h \), \( z_2 = 0 \), \( v_1 = 0 \)

\[ 0 + \frac{1}{2} \rho(0)^2 + \gamma(h) = 0 + \frac{1}{2} \rho v_2^2 + \gamma(0) \]

\[ v_2 = \sqrt{2gh} \]

3. Insert \( v_2 \), determined from the Bernoulli eq. into the continuity eq.
4. (Bernoulli eq. & Continuity) Water flows through a pipe reducer as shown. The static pressures at (1) and (2) are measured by the inverted U-tube manometer containing oil of specific gravity, $SG$, less than one. Determine the manometer reading, $h$.

**Sol** 1. *U-tube manometer reading*

From (1) → (A) → (B) → (C) → (2)

$$p_1 - \gamma(z_2 - z_1) - \gamma l - \gamma h + \gamma_{oil} h + \gamma l = p_2$$

where $\gamma_{oil} = SG\gamma$

or

$$p_1 - p_2 = \gamma(z_2 - z_1) + (1 - SG)\gamma h$$

2. $p_1 - p_2$: To be determined from the Bernoulli eq.

$$p_1 + \frac{1}{2} \rho v_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho v_2^2 + \gamma z_2$$

or

$$p_1 - p_2 = \gamma(z_2 - z_1) + \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$$

3. By comparing two eqs.,

$$(1 - SG)\gamma h = \frac{1}{2} \rho [v_2^2 - v_1^2] = \frac{1}{2} \rho v_2^2 \left[1 - \left(\frac{v_1}{v_2}\right)^2\right]$$

4. $v_2^2 - v_1^2$: To be determined from the continuity eq. (Confined flow)

$$Q_1 = A_1 v_1 = \frac{\pi}{4} D_1^2 v_1 = \frac{\pi}{4} D_2^2 v_2 = A_2 v_2 = Q_2 \quad \rightarrow \quad \frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{D_2^2}{D_1^2}$$

5. We need one more condition to determine $v_2$, (e.g. $Q = A_2 v_2$)
5. (Bernoulli eq. & Continuity & Pitot tube)

Two Pitot tubes and two static pressure taps are placed in the pipe contraction shown. The flowing fluid is water, and viscous effects are negligible. Determine the two manometer readings, \( h \) and \( H \).

Sol) (1) Manometer reading \( h \) between (1) & (2)

From (1) → (A) → (B) → (2)

\[
p_1 + \rho \gamma_1 - S G \gamma h - \gamma (l_1 - h) = p_2 \quad \text{where } \gamma_{oil} = S G \gamma
\]

or

\[
(1 - S G) \gamma h = p_2 - p_1 \quad (\ast \text{ Independent to } l_1)
\]

* \( p_2 - p_1 \): To be determined from the Bernoulli eq.

\[
p_1 + \frac{1}{2} \rho v_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho v_2^2 + \gamma z_2
\]

where, \( z_1 = 0, \ z_2 = 0, \ v_1 = 0 \ & \ v_2 = 0 \) (Stagnation points!!)

\[
\therefore \ p_2 - p_1 = 0 \quad \text{and thus } \ h = 0
\]

(2) Manometer reading \( H \) between (3) & (4) [(3) → (C) → (D) → (4)]

\[
p_3 - \gamma(l_3 - \frac{1}{12}) + \gamma_{air} \sqrt{H} + \gamma(l_3 - H) = p_4 \quad \text{where } \gamma_{air} : \text{Negligible}
\]

or

\[
\gamma(H - \frac{1}{12}) = p_3 - p_4
\]

* \( p_3 - p_4 \): To be determined from the Bernoulli eq. & the continuity eq.

\[
p_3 + \frac{1}{2} \rho v_3^2 + \gamma z_3 = p_4 + \frac{1}{2} \rho v_4^2 + \gamma z_4
\]

where, \( z_3 = 3/12 \ \text{ft}, \ z_4 = 2/12 \ \text{ft}, \ v_3 = 2 \ \text{ft/s}, \ v_4 = \frac{A_3}{A_2} v_3 \)