ELECTROMAGNETIC INDUCTION AND INDUCTANCE

OBJECTS
♦ To study electromagnetic induction;
♦ To study the DC transient behavior of a Resistance-Inductance (RL) circuit;
♦ To study the AC behavior of a RL circuit.

EQUIPMENT
A sine wave/square wave signal generator, digital multimeters (DMM), a pair of coils, a bar magnet, a galvanometer, and an oscilloscope are provided. See the Equipment Reference document for detailed information.

The galvanometer was not used previously and is not described in the Equipment Reference.

GALVANOMETER
A galvanometer is an instrument for detecting and measuring very small currents. Because it is essentially a sensitive ammeter, it is connected in series in the circuit whose current is to be measured.

THEORY (in addition to the information presented in the lecture)

DC BEHAVIOUR OF A SERIES LR CIRCUIT
The behaviour of an inductance in a circuit is determined by Faraday’s Law of Induction, which states that the emf generated across the inductor is proportional to the rate of change of the magnetic flux through the inductor. Since the magnetic flux is proportional to the current, the above statement is equivalent to saying that the emf, or voltage $V_L$ across the inductor, is proportional to the rate of change of current in the inductor,

$$V_L \propto \frac{dI}{dt}.$$  

The proportionality constant required to make this an equation is called the inductance, $L$, of the inductor:

$$V_L = L \frac{dI}{dt}$$

The inductance has units $\frac{V}{A \cdot s} = \frac{V \cdot s}{A}$, which is the SI unit the Henry, H. An inductance has a value of 1 H if one volt potential difference occurs across it when the current is changing at a rate of 1 A/s.

The other principle determining the polarity of the potential difference $V_L$ is Lenz’s Law, which states that the induced voltage $V_L$ has a polarity so as to oppose the change in the current.
For example, if the current is increasing \( \left( \frac{dI}{dt} \right) \) positive), the inductor will act like a battery of voltage \( L \frac{dI}{dt} \) which sends out current opposite to the incoming current (opposing the increase).

If the current is decreasing \( \left( \frac{dI}{dt} \right) \) negative), the inductor will act like a battery of voltage \( L \frac{dI}{dt} \) which sends out current which tries to increase the incoming current back to its original value (opposing the decrease).

Consider the behaviour of a series LR circuit when a square wave voltage is applied.

The voltage signals (waveforms) across the inductor and resistor can be calculated using Kirchhoff’s Voltage Law (KVL), Faraday’s Law, and Ohm’s Law.

From KVL, for the square wave generator connected to an inductor and resistor in series,

\[
V_{\text{sq wave}} = V_L + V_R
\]

Consider the ON half of the square wave:

\[
V_{\text{sq wave}} = V_o \text{ (constant DC voltage)}
\]

Therefore

\[
V_o = V_L + V_R
\]

\[
V_o = L \frac{dI}{dt} + IR
\]
When integrated in exactly the same way as was done for the RC circuit, the solution to this equation is

$$I(t) = \frac{V_o}{R} (1 - e^{-Rt/L}) = \frac{V_o}{R} (1 - e^{-t/\tau})$$

where $\tau = L/R$ is the time constant.

The inductor voltage is given by

$$V_L(t) = L \frac{dI(t)}{dt} = V_o e^{-t/\tau}$$

and

$$V_R(t) = RI(t) = V_o (1 - e^{-t/\tau})$$

For the other half of the square wave cycle, when the output is ‘SHORTED’, the waveforms are obtained in the same way.

Now, $V_{\text{sq wave}} = 0$ (circuit behaves as if a wire were connected across the input terminals)

Therefore

$$0 = V_L + V_R$$

Solving as before,

$$I(t) = \frac{V_o}{R} (e^{-t/\tau})$$

$$V_L(t) = L \frac{dI(t)}{dt} = -V_o e^{-t/\tau}$$

$$V_R(t) = V_o e^{-t/\tau}$$

**FREQUENCY DEPENDENT METHOD FOR MEASURING INDUCTANCE**

**INDUCTOR:** Voltage leads current by 90°

Voltage and current are related by the equation

$$V_L = X_L I$$

where $X_L = \omega L = 2\pi f L$ is the inductive reactance of the inductance $L$ at frequency $f$.

The average power dissipation is 0.

Consider the circuit with an inductor and resistor in series fed by an ac signal generator.
By Ohm's Law for inductors, the inductor voltage $V_L$ is given by

$$V_L = X_L I = 2\pi f L I$$

This can be rewritten as

$$V_L = (2\pi f L) I$$

which is a linear relation between the inductor voltage, $V_L$, and the frequency, $f$, provided that the inductance, $L$, and the current, $I$, are constant.

An experimental plot of $V_L$ versus $f$ with $I$ maintained constant will give a straight line whose slope can be measured.

The value of the inductance is then

$$L = \frac{\text{slope of graph}}{2\pi f}$$

where $I$ is the value of the constant current maintained through the circuit.

**EXPERIMENT**

**A. ELECTROMAGNETIC INDUCTION**

Connect the 300 turn coil to the galvanometer as shown below. Be sure that the coil terminal labelled “B” is connected to the galvanometer red terminal. Connect the rheostat so that its resistance can be adjusted.

The bar magnet can be used to produce a varying magnetic flux through the coil.

If an emf is induced in the coil, the galvanometer will show the magnitude and direction of the resulting induced current.
Use the circuit shown above to examine the validity of Faraday’s Law of Induction by recording the magnitude and polarity of the galvanometer deflection as the magnetic flux through the coil is changed.

In particular, examine the effects of varying:

- the speed with which the magnet is moved;
- the direction in which the magnet is moved;
- the direction of the magnet’s field;
- the number of turns in the coil.

Record your observations in a manner similar to the following:

<table>
<thead>
<tr>
<th>Coil (turns)</th>
<th>Magnitude &amp; Polarity of Galvanometer Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North in</td>
</tr>
<tr>
<td>100 or 300</td>
<td></td>
</tr>
<tr>
<td>200 or 600</td>
<td></td>
</tr>
</tbody>
</table>

Dependence on speed of magnet motion:

Dependence on # of coil turns:

**SERIES RL CIRCUIT (SQUARE WAVE SOURCE)**

1. Connect the circuit as shown, using a 2.2 kΩ resistor from your resistor board (BC or DE) and the inductor from the LC board. Set the square wave generator frequency at 1000 Hz.

Connect the oscilloscope to the circuit so that Channel 1 displays the square wave voltage (the signal being input to the circuit) and Channel 2 displays the resistor voltage.

If no signal appears in scope and a yellow Armed message appears in the middle of the screen at the top, this means that the scope is not triggering. Try adjusting the Horizontal (Time) scale to a slower (longer time base) setting until a green Trig’d message appears (and hopefully the signal appears as well!).
Set Trigger Coupling to AC:
- Press Menu button in Trigger section
- Press more menu button (at bottom)
- Press Coupling menu button
- Highlight and Select AC by using the Multipurpose knob

Display of input channels is toggled ON/OFF by using the 1 and 2 buttons.

In your lab notebook, draw axes for a voltage vs. time graph. Allow room for negative as well as positive voltages. NOTE: This graph should only occupy about a third of a page (not a full page).

From your observations of the oscilloscope display, sketch both the square wave and resistor voltage waveforms on the graph. Does the resistor waveform agree with the theoretically predicted waveform?

In addition to viewing the Square (Ch1) and Resistor (Ch2) signals, use the Math button to also view the difference of these two signals:
- Press the pink M button under Math
- Press the Operation button and use the Multipurpose knob to highlight and select subtraction
- Use the Sources button to select Ch1 – Ch2
- (Display of the Math waveform is toggled ON/OFF using the pink M button)

Sketch the subtraction waveform on the same graph as the square and resistor waveforms.

As was done for the RC circuit in Experiment 2, measure as accurately as possible the time constant for both the rise and fall of the resistor voltage.

Measure the charging and discharging time constants by measuring the time for the resistor voltage to rise to 0.632 of the applied voltage and measure the discharging time constant by measuring the time for the resistor voltage to fall to 0.368 of the applied voltage:

- Adjust the Function Generator DC Offset and Output Level so that the square wave is all positive with an amplitude of 8 V
- Set Ch1 and Ch2 Scales to 2.00 V (so that sq. wave is 4 cm high)
- Expand Horizontal (time) scale so that one charge or one discharge segment fills the screen horizontally
- Press the Cursor button and use the menu buttons to select Type:Time and Source:Ch2
- Position Cursor 1 at the start of the charge or discharge
- Position Cursor 2 at 5.06 V (0.632 x 8.00 V) for charge or 2.94 V (0.368 x 8.00 V) for discharge
- The middle panel on the screen under Cursor will show a time difference that is the time constant (you can also check that the voltage difference is correct)
These time measurements should be the same, and should equal the theoretical time constant, $\tau = \frac{L}{R}$.

2. Now interchange the inductor and the resistor in the network, as shown:

Use Channel 1 of the oscilloscope to display the square wave and use Channel 2 to display the inductor voltage. To clearly show the full inductor voltage and its relation to the applied square wave voltage, you will need to set the 0 voltage location to the middle of the grid and adjust your VOLT/DIV scales accordingly.

Carefully compare the inductor voltage waveform displayed on the scope with the Square-Resistor subtraction waveform that you’ve drawn on your graph. Do the inductor waveform and the subtraction waveform have the same shape?

Do you expect the inductor waveform and the subtraction waveform to have the same shape? Discuss the answer to this question in the context of Kirchhoff’s Voltage Law (voltage applied = sum of the voltage drops around a closed loop)?

**SERIES RL CIRCUIT (SINE WAVE SOURCE)**
1. Measure the resistance of the carbon resistor provided.
2. Connect the ac signal generator to the series combination of the inductor and the provided carbon resistor.
3. Keeping the current constant (by maintaining a resistor voltage of 1.00 V by adjusting the audio generator output amplitude as necessary), measure the inductor voltage, $V_L$, for frequencies from 500 to 2000 Hz.

**ANALYSIS**

**ELECTROMAGNETIC INDUCTION**

To aid in analyzing the response of the galvanometer, note that the polarity of the galvanometer deflection indicates the polarity of the terminal connected to the red galvanometer post. Also, remember that current flows from positive to negative and that the coil is the source of emf in the circuit (so current flows out of the positive coil terminal, through the circuit, and back into the negative coil terminal).

The coils are wound as shown below – when viewed from above, the winding is counterclockwise from terminal A to terminal B.

When terminal B is positive, indicating that current is flowing out of B, the induced magnetic field has a North pole at the top of the coil. When terminal B is negative, indicating that current is flowing into B, the induced magnetic field has a South pole at the top of the coil.

Based on the previous paragraph, interpret your deflection polarity observations in terms of the predictions of Lenz’s Law. Is there agreement? Discuss.

What does Faraday’s Law predict for the dependencies of deflection magnitude on magnet speed and number of coil turns? Do your observations agree with these predictions?

**SERIES RL CIRCUIT (SQUARE WAVE SOURCE)**

1. Does the shape of the resistor waveform agree with the theoretically predicted waveform?

2. Given that the theoretical value of the time constant is $\tau = \frac{L}{R}$, calculate the value of the inductance.

3. By comparing the shapes of the inductor and resistor waveforms on your graph, determine the approximate shape of the SUM of the inductor and resistor voltage waveforms. How does this shape compare with that predicted by Kirchhoff’s Voltage Law (voltage applied = sum of the voltage drops around a closed loop)?

**SERIES RL CIRCUIT (SINE WAVE SOURCE)**

1. Use Ohm’s Law to calculate the (constant) current.

2. Plot the graph of inductor voltage versus frequency and use the slope of the graph to calculate the inductance.