A: GENERAL INSTRUCTIONS

Safety in the Laboratory

The safety issues related to the equipment and procedures used in the Physics 117 laboratory are as follows:

- thermal hazards due to use of mercury light source (experiment L11)
- electrical hazards, including high current and high voltage

General Safety Policies

- **No food or drink is to be consumed in the laboratory, nor be on the laboratory benches.** Liquids brought into the lab room must be kept in sealed containers, which are not to be opened in the lab room and which must be placed on the floor (i.e. below the lab bench).

- **Follow all instructions**
  In addition to the instructions in this laboratory manual, follow all verbal and written instructions provided by the instructional staff.

Come Prepared to the Laboratory

A proper understanding of your first year physics class requires a combination of studying the lecture material, solving problems, and performing experiments in the laboratory. An attempt has been made to relate the experimental work directly to the lecture material. Due to space and equipment limitations, however, experimental work may, in some cases, have to be carried out before the topic has been covered in the lectures. To compensate for this, before coming to the laboratory you should carefully read the sections of your text which are covered in the assigned experiment. If the material has been taken in lectures, the lecture notes should be studied as well. After having done this preparatory reading, carefully review the object, theory and procedure outlined in the lab manual so that you have an understanding of the purpose of the experiment, what data will be required to accomplish that purpose, and how that data will be collected. Use the material presented in this manual to plan in advance the way the experimental work should be organized.

Partners

Although students are allocated partners during the first laboratory period of the term, the laboratory reports should be written independently. This does not imply that partners should not cooperate; time and effort can be saved by working together in sharing the numerical calculations, checking results, and discussing the theory and performance of each experiment. Partners should share the work so that both partners understand the whole of the experiment and become familiar with the equipment. Partners may, and should, discuss their conclusions and sources of error with each other, BUT they must each write these sections in their own words.

Laboratory Assignments

A schedule of assignments, indicating who which experiments, when, and where, will be posted outside rooms 112 and 117, before the second laboratory session.

Grades

Marks obtained for the laboratory work account for 15% of the final grade, unless otherwise indicated by the professor of the class.
Materials

Each student should bring to the laboratory an ACCO folder and 3-hole-punched sheets of ¼" grid paper, an inexpensive set of geometrical instruments, and an approved calculator (Texas Instruments TI-30X series or Hewlett Packard HP 10s or HP 30S). Specific instructions about the materials required for the laboratory classes will be given in the first meeting of the class.

Absenteeism

Students who are absent from a laboratory period for a justifiable reason (illness, Huskie Athletics...) may be able to arrange with the laboratory instructor to perform the experiment at another time. Documentation explaining the absence should be presented to the laboratory instructor whenever possible.

Assistance in the Laboratory

The demonstrator in the laboratory is there to assist you. When something arises which you do not understand, and which cannot be resolved with the help of your partner, please consult the demonstrator.

BEFORE LEAVING THE LABORATORY, RETURN EQUIPMENT TO ITS ORIGINAL LOCATION SO THAT THE NEXT GROUP WILL NOT START THEIR EXPERIMENT AT A DISADVANTAGE. DISCONNECT ELECTRICAL CONNECTIONS. TURN OFF GAS, WATER AND AIR USED IN THE EXPERIMENT.
B: LABORATORY LOGS

One of the aims of this laboratory is to teach you the methods that professional physicists have found satisfactory for recording the performance of experiments. Two criteria should be met by a satisfactory laboratory log:

1. A person with an educational background similar to yours should be able to easily follow through the experiment by reading the lab manual and your log, and if desired should be able to easily perform the experiment, obtaining results similar to yours.

2. The results of your study should stand out clearly and the conclusions drawn from these results should follow logically.

A suggested approach follows on the organization of your work to meet the above criteria. Details, such as correct procedures for drawing graphs, calculating errors, etc. are covered in later sections.

A log book format, rather than a formal lab report format will be used. The emphasis is on whether or not you understood and properly performed the experiment. The information contained in the log book should clearly indicate how the experiment was performed, what measurements were made, how these measurements were analyzed, and what conclusions were drawn. It is essential that you read the laboratory manual before coming to the lab so that you have a basic understanding of what you will be doing.

The log for each experiment will contain the following sections:

**Heading**
- title of experiment, date experiment performed, your name, partners’ names

**Object**
- one or two sentences describing purpose of experiment

**Experiment**

**Data Collection**
- sketch of equipment, notes outlining procedure steps, and data, for each part of the experiment
- a brief note should accompany the equipment sketch, e.g. “The spark timer is used to record a trace of the position of the cart at equally spaced time intervals.” These notes are to be done at the time the measurements are made and are to be very brief. If the result of a particular step in the procedure is the measurement of a single quantity, the note can be incorporated in the recording of the measurement. For example, “distance between legs of air track, measured with tape measure: $L = 153.2 \pm 0.4 \text{ cm}$” would be a suitable note. If a number of measurements are made by repetition of a single procedure, the note forms a description of the corresponding table of values, e.g. “A ruler was used to measure the displacement, $\Delta x$, of the cart in 0.200 s time intervals at various points along the spark trace.”
- data should be tabulated whenever possible
- tables must be titled, and as much information as possible is to be put in the column headings rather than included with each entry. The heading of a column contains a one- or two-word descriptive label, the symbol for the quantity being tabulated, the units, the power of ten if scientific notation is being used, and the experimental error (if constant for
all values in column). Individual column entries will be numbers or numbers ± errors. See Table 1 further in this introduction for an example of a proper table.

- data must be recorded, with their errors and units, directly into the log book (transfer of measurements recorded on scrap paper wastes time and can introduce mistakes)
- incorrect data are to be neatly crossed out with a single line in such a way that they are still legible

**Analysis**

- this section involves manipulation of the raw observational data by graphing, making numerical calculations, or in some other way specified in the instructions
- sample calculation (and error calculation if required) for each different equation
- units must be carried through calculations
- use a separate, complete page for each graph
- analysis results should be presented in tabular form whenever possible
- if possible, compare experimental results with theoretical or accepted values. The primary criterion to be used when comparing values is whether or not the values agree within experimental error (i.e. do the error ranges overlap?). Although it is not the preferred comparison criterion, percentage difference between experimental and accepted/theoretical values may be used on occasion.

**Conclusion**

- state results (with experimental error) and state whether or not there is agreement within experimental error with theoretical or accepted values
- **must include a discussion of the physical concepts examined in the experiment**

**Sources of Error**

- state as many factors as you can think of which might have affected the outcome of the experiment, and which were not accounted for in any way
- explain how each of the above factors would be expected to affect your calculated results, and whether the effects would be large or small
- random measuring error need not be mentioned (it is usually accounted for in error calculations)
- the possibility of incorrectly calibrated measuring instruments should not be mentioned unless there is a particular reason for doing so. The same holds for the possibility of incorrect calculations.

Point-form may be used for the written sections of the log, but you must ensure that the meaning of your comments is clear. **Written parts of the log entries, and data, are to be done in pen; pencil may be used for diagrams and calculations.** For work done in pen, corrections are to be made by drawing a single line through the incorrect values and/or statements, then writing the correct value/statement.
LOG BOOK STYLE SAMPLE

The following is intended to show by example the style that is expected for your laboratory logs. The example is for the period measurement done in an experiment examining the simple harmonic motion of a mass on a vertical spring. The sample report will be written as if the period measurement comprised a complete experiment.

5 MAY 95        M19 – HOOKE’S LAW AND SIMPLE HARMONIC MOTION        BRIAN ZULKOSKEY

Object: to measure the period of the oscillations of a vertical mass-spring system

Experiment:

A 200 g load was placed on the pan and the oscillation period, \( T \), was measured five times using a photogate timer.

Data:

<table>
<thead>
<tr>
<th>Trial</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period, ( T ) ( (\pm 0.0001 \text{ s}) )</td>
<td>0.5731</td>
<td>0.5750</td>
<td>0.5737</td>
<td>0.5745</td>
<td>0.5739</td>
<td>0.57404 ± 0.00057</td>
</tr>
</tbody>
</table>

Analysis:
Calculation of Average Period:

\[
T_{\text{ave}} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5}
\]
\[ T_{\text{ave}} = \frac{0.5731s + 0.5750s + 0.5737s + 0.5745s + 0.5739s}{5} \]

\[ T_{\text{ave}} = 0.57404s \]

Calculation of error in Average Period (Average Deviation of Period values):

\[ \delta T_{\text{ave}} = \text{ave.dev.} = \frac{|T_1 - T_{\text{ave}}| + |T_2 - T_{\text{ave}}| + |T_3 - T_{\text{ave}}| + |T_4 - T_{\text{ave}}| + |T_5 - T_{\text{ave}}|}{5} \]

\[ \delta T_{\text{ave}} = \text{ave.dev.} = \{|0.5731s - 0.57404s| + |0.5750s - 0.57404s| + |0.5737s - 0.57404s| + |0.5745s - 0.57404s| + |0.5739s - 0.57404s|\} ÷ 5 \]

\[ \delta T_{\text{ave}} = \text{ave.dev.} = 0.00057s \]

\[ T_{\text{ave}} = 0.57404 ± 0.00057s \]

**Conclusion:**
The average period of the oscillating vertical mass-spring system was determined to be 0.57404 ± 0.00057 s. *(In a regular experiment you would normally compare your experimental result to a theoretical or accepted value and discuss the significance of the result of the comparison.)*

**Sources of Error:**
- it was difficult to avoid sideways motion of the mass-spring system, which would lengthen the period (moderate effect)
- air resistance will damp the motion (small effect)
In order to understand a physical problem, one usually studies the dependence of one quantity upon another. Whereas sometimes such a dependence is obtained theoretically, at times it must be arrived at experimentally. The experimental determination involves obtaining experimentally a series of values of one quantity corresponding to the various arbitrary values of the other and then subjecting the data to some kind of analysis. One of the most convenient and useful means of treating the experimental data is by graphical analysis.

A graph shows the relation between the two quantities in the form of a curve. Suppose we are interested in looking at the relation between the volume of a piece of iron and its mass. If we make measurements on pieces of iron, we find that 1 cm$^3$ has a mass of 7.87 g, 2 cm$^3$ has a mass of 15.74 g, 3 cm$^3$ has a mass of 23.61 g, and so on. This kind of relation in which doubling the volume doubles the mass, tripling the volume triples the mass, is called direct proportion. In Physics you will encounter many cases of such relations. In describing this relation we say:

(i) Mass ‘is directly proportional to’ volume of iron or mass ‘varies directly as’ the volume of iron.

(ii) Mathematically we can write the relation as $M \propto V$, where $M$ is the mass of a piece of iron and $V$ is its volume and the symbol $\propto$ means ‘is proportional to’. If we have two different volumes of iron, $V_1$ and $V_2$, their masses $M_1$ and $M_2$ may be expressed as

$$\frac{M_1}{M_2} = \frac{V_1}{V_2}$$

Another form of this relation expresses the fact that when mass and volume are related by direct proportion, they have a constant ratio. Thus

$$\left( \frac{M}{V} \right) \text{ of one sample} = \left( \frac{M}{V} \right) \text{ of another sample} = k$$

The constant $k$ is called the proportionality constant. In our example of iron

$k = 7.87$ g for each cubic centimetre

We now express this relation as an equation for any piece of iron:

$$\left( \frac{M}{V} \right) = k \text{ or } M = kV$$

where the value of $k$ as determined experimentally is 7.87 g/cm$^3$.

An equation established in this way is an empirical equation.

We can illustrate this relation between mass and volume for iron by a graph. In trying to study the mass of a piece of iron as a function of its volume, we chose certain values for the volume and determined the corresponding values for the mass. In such a situation volume is said to be the independent variable and the mass is the dependent variable. Generally, the independent variable should be the variable that is altered in regular steps in the experiment, and the dependent variable is the quantity which is measured for each regular step of the independent variable. The independent variable (in our case the volume of a piece of iron) is plotted along the $x$ axis and the dependent variable (in our case, the mass of the piece of iron) is plotted along
the $y$ axis. To plot the data we must choose scales – one for the vertical direction, marking off some suitable number of grams for each vertical division of the paper, and one for the horizontal direction, marking off volumes in cm$^3$. Now we can place a point on the graph for each pair of values that we know.

<table>
<thead>
<tr>
<th>Volume, $V$ (cm$^3$)</th>
<th>Mass, $M$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.87</td>
</tr>
<tr>
<td>2</td>
<td>15.74</td>
</tr>
<tr>
<td>3</td>
<td>23.61</td>
</tr>
</tbody>
</table>

Note that a straight line can be drawn through the data points: the graph is linear. The ratio $\left( \frac{M}{V} \right)$ is the same for all points on the line, i.e. as already noted, $\left( \frac{M}{V} \right) = k$ or $M = kV$ where $k$ is a constant. As seen here and discussed in the next section, $M = kV$ is the equation of a straight line passing through the origin if $M$ and $V$ are plotted against each other. $k$ is the slope of the line.

General Linear Relationship

A straight line plot of $y$ versus $x$ indicates that the relationship is linear and is of the form

$$y = mx + b$$

where $b$ is the intercept on the $y$-axis (the value of $y$ when $x = 0$) and $m$ is the slope of the line.

As an example, consider an examination of the relation between the strain ($x$) produced in a spring as a function of the applied force ($F$). A linear relation implies

$$F = kx + b$$

where $k$ and $b$ are constants. For this example refer to the data in Table 1.

<table>
<thead>
<tr>
<th>Force, $F$ (± 0.05N)</th>
<th>Average Strain, $x$ (± 0.1 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19</td>
<td>3.0</td>
</tr>
<tr>
<td>0.40</td>
<td>6.2</td>
</tr>
<tr>
<td>0.71</td>
<td>9.6</td>
</tr>
<tr>
<td>1.00</td>
<td>12.6</td>
</tr>
<tr>
<td>1.26</td>
<td>16.0</td>
</tr>
<tr>
<td>1.50</td>
<td>19.2</td>
</tr>
</tbody>
</table>
Graph 1: Force vs. Average Strain to determine Spring Constant

\[ \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.40 \text{ N} - 0.40 \text{ N}}{17.8 \text{ cm} - 5.8 \text{ cm}} = \frac{1.00 \text{ N}}{12.0 \text{ cm}} = 0.0833 \text{ N/cm} = 8.33 \text{ N/m} \]

\[ \text{max slope} = \frac{y_4 - y_3}{x_4 - x_3} = \frac{1.55 \text{ N} - 0.14 \text{ N}}{19.2 \text{ cm} - 3.0 \text{ cm}} = \frac{1.41 \text{ N}}{16.2 \text{ cm}} = 0.0870 \text{ N/cm} = 8.70 \text{ N/m} \]

\[ \delta(\text{slope}) = \text{max slope} - \text{slope} = 8.70 \text{ N/m} - 8.33 \text{ N/m} = 0.37 \text{ N/m} \]

\( (x_1, y_1) = (5.8 \text{ cm}, 0.40 \text{ N}) \)

\( (x_2, y_2) = (17.8 \text{ cm}, 1.40 \text{ N}) \)

\( (x_3, y_3) = (3.0 \text{ cm}, 0.14 \text{ N}) \)

\( (x_4, y_4) = (19.2 \text{ cm}, 1.55 \text{ N}) \)
The experimentally obtained points usually do not lie exactly on a straight line; this is expected, because experimental data are never exact. In a straight line plot of experimental results, the straight line goes above some points and below others and represents an honest attempt on the part of the plotter to show the trend of the data. See Graph 1.

If the plot of \( y \) versus \( x \) yields a straight line, the relationship is linear and is of the form

\[
y = mx + b
\]

If \( m = 0 \), \( y \) does not depend on \( x \). If the plot is a straight line passing through the origin, \( b = 0 \).

When drawing graphs, the following rules are to be observed in order that the graphs neatly and clearly represent the experimental data.

1. Draw the axes. Axes are not normally placed along the boundaries between the graph paper and the margin although in some cases it may be necessary to do so because of the extent of the graph. Axes are usually drawn one or two large divisions in from the left and up from the bottom margin respectively. Label each axis with the variable being plotted (e.g. mass, period, time, etc.) and the units.

2. Mark the scales along the axes. The following points should be considered in choosing the scale:
   a) the graph should be easy to read and it should be possible to easily interpolate values without the need for a calculator. Make each large division equal to 1, 2, 4, 5 or 10 units. Do NOT use scales of 3, 6, 7, 9, ... units per division;
   b) the resultant curve should fill the whole graph and should not be confined to a small area of the graph paper;
   c) the geometrical slope of the curve (if it is a straight line) should be approximately unity;
   d) the point \((0, 0)\) should not appear on the scales unless the point \((0, 0)\) is a significant point on the graph.

3. Plot the points carefully. Mark each point so that the plotted point stands out clearly. When two sets of different data are plotted on the same set of axes use different symbols (e.g. circles and crosses) for each set of data.

4. Fit a curve to the plotted points. Most of the graphs drawn will be for the purpose of illustrating a law or determining a relationship. Hence it is reasonable to assume that the graph should be a uniformly smooth curve, possibly a straight line. Owing to the limits of experimental accuracy not all the plotted points will be exactly on the smooth curve or straight line. Use a transparent French curve or straight edge and draw a curve through the plotted points so that:
   a) the curve is smooth;
   b) the curve passes as close as possible to all the data points;
   c) when taken in moderately sized groups, as many points fall on one side of the curve as on the other.

Note that the curve need not pass through any one point, and certainly need not pass through the end points as these end points are usually obtained at the limits of the accuracy of the
measuring instrument. Use dashes to show the extrapolation of the curve past the range of the data points.

5. Give the completed graph a descriptive title (what is being plotted and why). This should be placed in a position where it does not interfere with the curve.

6. If a slope calculation is required it should be done on the graph page or on the page immediately following the graph page. This reduces the possibilities of error in slope calculations and makes interpretation of the graph easier. A LARGE triangle should be drawn on the graph with vertices \((x_1, y_1)\) and \((x_2, y_2)\) as shown in Graph 1. The vertices are then used to calculate the slope according to the formula:

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}
\]

Data points should not be used in slope calculations.

The experimental error in the slope can be calculated by performing an error calculation (discussed in a later section) on the slope equation.

7. It is sometimes desirable to indicate the experimental error in the plotted data points (see the section Discussion of Errors). This is done with ERROR BARS, which are shown in Graph 1. Vertical error bars are drawn by going above and below a data point a distance, determined by the vertical scale, corresponding to the error in the \(y\)-value. Horizontal error bars are drawn by going on either side of the data point a distance corresponding to the error in the \(x\)-value.

When error bars are drawn, the experimental error in the slope of a straight line is determined by one of the following methods:

**Method 1**

a) as already discussed, the best-fit straight line is drawn and its slope is calculated;

b) another line is drawn on the graph, this EXTREME-FIT line is the line passing through or near the error bars of all the data points and having the maximum or minimum possible slope;

c) the error in the best-fit slope is then taken as the absolute value of the difference between the extreme-fit slope and the best-fit slope.

**Method 2**

a) both the maximum-slope and the minimum-slope EXTREME-FIT lines are drawn on the graph;

b) the average slope is taken as the average of the slopes of the maximum-fit and minimum-fit lines;

c) the error in the average slope is taken as half the difference between the maximum-fit slope and the minimum-fit slope.

\[
\text{avg slope} = \frac{(\text{max} + \text{min})}{2}; \quad \delta(\text{avg slope}) = \frac{(\text{max} - \text{min})}{2}
\]
D: DISCUSSION OF ERRORS

Accuracy of Physical Measurements

Whether an experiment is of a precision type in which the answer is the magnitude of a quantity or of a qualitative nature in which the main aim is to substantiate a qualitative conclusion, it always involves taking measurements. Every measurement of a physical quantity is uncertain by some amount. When an observation or a result is recorded, one may ask the question - how precisely do we know this quantity? What we are asking for is a statement of some range of confidence in the quoted value or an expression of the uncertainty, sometimes called the error, in the quantity. By the word error we do not mean mistake but rather the uncertainty in the quantity.

There are three types of errors that may occur in experimental work:

1. Blunders or Mistakes on the part of the observer in reading the instruments or recording observations can be avoided by the exercise of reasonable care and therefore need no discussion. Some simple rules that will reduce the possibility of such mistakes are:
   a) consider whether or not the reading taken is reasonable. Is it of the order of magnitude expected?
   b) record the reading immediately and directly in the laboratory notebook. Do not attempt to remember it even for a few minutes nor record it on a loose sheet;
   c) whenever possible have your partner check your reading.

2. Systematic Errors refer to uncertainties that influence all the measurements of a particular quantity equally. These errors may be inherent in the calibration of a particular instrument or in the particular design of the apparatus used. For example, the reading on a certain thermometer may be consistently too high because the thermometer was incorrectly calibrated. Systematic errors can usually be eliminated either by calibrating the particular instrument under conditions similar to those of the experiment or by applying a correction term in the calculations. In those cases where it is known to be necessary to apply a correction for a systematic error in the experiments performed in this laboratory, the instructors will give the details of the procedure to be followed.

3. Experimental or Random Errors remain even when all mistakes and systematic errors have been avoided or accounted for. Thus when a given measurement is repeated the resulting values, in general, do not agree exactly. No one has so far designed a perfect experiment: imperfections in our senses; in the instruments we use; and chance variations in conditions assumed to remain constant all combine to cause uncertainties in our measurements. These random errors are unavoidable and their exact magnitude cannot be determined.

A good example of these random deviations or accidental errors is provided by the shot pattern obtained when an accurately zeroed and solidly clamped rifle is fired at a target. The pattern of fire will show a scatter of shots about the centre of fire, the number of shots in a given annular ring of constant width being progressively less as the distance from the centre of fire increases. The same effect may be demonstrated by repeatedly dropping a dart or sharp-pointed pencil vertically onto a mark on a piece of paper. The scatter of the shots about the aiming mark is analogous to the spread of experimentally obtained values about their mean value, due to accidental errors.
It is often difficult to determine the magnitude of systematic errors in an experiment. For example, in the case discussed of a systematic error introduced by a poorly calibrated thermometer one doesn't know if the error is 1° or 10° unless the thermometer is compared to a reliable thermometer. In the case of random errors, however, the data taken in the experiment is enough to indicate how large these errors are.

A common way of approximating the magnitude of random errors is the use of significant figures. For a more reliable estimate of these errors, statistical methods and error calculations are employed.
Numerical quantities have a different meaning in physics than they do in mathematics. The term ‘significant figures’ refers to the fact that while to a mathematician numbers are exact quantities, to a physicist a number is often a measured quantity and therefore inexact. A rough approximation of the uncertainty or experimental error in a number is indicated by the physicist by only writing those figures of the number in which he/she has confidence, i.e. by only writing the figures that are significant. For example, to a mathematician, 16 is the same as 16.0 or 16.00, but to a physicist, if \( x \) is a length measurement then \( x = 16 \) cm means \( 15.5 \text{ cm} < x < 16.5 \text{ cm} \); \( x = 16.0 \) cm means \( 15.95 \text{ cm} < x < 16.05 \text{ cm} \); and \( x = 16.00 \) cm means \( 15.995 \text{ cm} < x < 16.005 \text{ cm} \).

Note that zeros used as place holders between the decimal point and the number are not significant and that zeros after the number may or may not be significant. To avoid this possible source of confusion very large and very small numbers are often written in scientific notation (also called power of ten notation). In this notation the number is written with one figure to the left of the decimal point and then multiplied by the appropriate power of ten.

e.g. \( .001804 = 1.804 \times 10^{-3} \) (4 sig. fig.)

These conventions regarding significant figures are illustrated in the following table:

<table>
<thead>
<tr>
<th>Two Significant Figures</th>
<th>Three Significant Figures</th>
<th>Four Significant Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>32.0</td>
<td>32.00</td>
</tr>
<tr>
<td>.0032</td>
<td>.00320</td>
<td>.003200</td>
</tr>
<tr>
<td>( 3.2 \times 10^4 )</td>
<td>( 3.20 \times 10^4 )</td>
<td>( 3.200 \times 10^4 )</td>
</tr>
</tbody>
</table>

When using significant figures in calculations remember that the result of a calculation cannot be expressed to a greater degree of accuracy than the numbers from which it is calculated. For example, consider a case in which the length of an object is 20.5 cm and the width is 14.23 cm, and we are interested in finding the sum of the length and width of the object. Representing unknown figures by a question mark, we may write the sum as:

\[
20.5?? + 14.23? = 34.7? 
\]

Notice that the sum of a known figure and an unknown one is an unknown figure. The result of this example can be generalized to the following statement: the result of an addition or subtraction calculation is rounded off to the same number of DECIMAL PLACES as the number with the least decimal places that is used in the calculation. (Note that before applying this rule all the numbers involved in the calculation must be expressed to the same power of ten if scientific notation is being used.)

Similarly, when multiplying or dividing, a result can have no more SIGNIFICANT FIGURES than does the number with the fewest significant figures that is used in computing the result.
Exercises on Significant Figures

1. How many significant figures does each of the following numbers have?
   a) $6.3 \times 10^3$
   b) .00370
   c) 6700
   d) $670.0 \times 10^3$

2. Round off to 2 significant figures and express in scientific notation:
   a) 3.95
   b) .02862
   c) 219
   d) 4326

3. Calculate to the appropriate number of significant figures and express in scientific notation:
   a) $63400 + 82$
   b) $480 \div .060$
   c) $379 - (6 \times 10^3)$
   d) $8900 \div 30$
   e) $290 + 6.7$
   f) $.030 - .003$
   g) $321 \times 3$

4. Suppose that you are instructed to draw a line ten centimetres long using an ordinary ruler (one which has millimetres marked on it). How will the length of the line be best expressed?
   a) $1 \times 10^1$ cm
   b) 10 cm
   c) 10.0 cm
   d) 10.00 cm
The ABSOLUTE error in a quantity is the estimate of the range of values to be expected due to experimental errors. The absolute error has the same units as the quantity and is denoted by the Greek symbol lower-case delta (\( \delta \)) in front of the symbol for the quantity. For example, if a length measurement is denoted by \( x \) then \( \delta x \) is the absolute error in this length. Experimental error can also be expressed relative to the quantity. This RELATIVE error is defined as the absolute error divided by the quantity. In symbols the relative error would be \( \delta x / x \) for the example mentioned above. If expressed as a percentage, the relative error becomes a PERCENTAGE error, and is calculated by \( (\delta x / x) \times 100\% \).

The correct use of significant figures gives a rough estimate of the magnitude of random errors involved in taking readings. A more precise and reliable method of determining random errors is to determine absolute errors by estimation or calculation and then to use these values in subsequent calculations. Generally speaking, calculated values can be no more precise than the individual measurements. In fact, the errors accumulate so that the calculated value is usually less precise than the measurements on which it is based. The errors in a calculated quantity can be determined from the errors in each of the directly measured quantities on which the calculation is based. Before proceeding to calculate the experimental error, however, we must decide what numerical value shall be given to the absolute error in any measurement. The following rules may be used for assigning absolute errors:

1. When a quantity is just measured once, the measured value is taken as being the ‘true’ value and the absolute error due to random causes is estimated as the MAXIMUM INSTRUMENTAL ERROR in the measuring instrument. The maximum instrumental error will usually be the least count of the instrument (the size of the smallest scale division). For example in measuring the length of an object with a meter stick, the error would be \( \pm 1 \) mm since 1 mm is the smallest scale division on the meter stick.

2. A more accurate estimate of error can be obtained when a quantity is measured several times. The arithmetic mean of the measurements is taken as the ‘true’ value and the average deviation from the mean is taken as the absolute error unless it is less than the maximum instrumental error as described in 1. The average deviation is given by

\[
\text{ave.dev.} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + ... + |x_n - \bar{x}|}{n}
\]

where \( x_1, x_2, ..., x_n \) are the individual measurements,

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}
\]

is the average of these readings,

and \( n \) is the number of measurements.
G: PROPAGATION OF ERRORS IN CALCULATIONS

In a number of cases the quantity of interest cannot be measured directly – it is calculated by using two or more directly measured quantities. When uncertain quantities are used in numerical calculations one usually wishes to know the error in the result. The simplest method, although not the most accurate, is to determine the maximum possible error.

MAXIMUM POSSIBLE ERROR METHOD

Bottom Line:
The maximum absolute uncertainty in the addition or subtraction of measured values is the sum of the absolute errors in the measured values. The maximum percentage (relative) uncertainty in the multiplication or division of measured values is the sum of the percentage (relative) errors in the measured values. The maximum percentage (relative) uncertainty in a measured value raised to a power is the product of the power and the percentage (relative) error in the measured value.

Details:
Addition and Subtraction

Consider the sum of two measurements, 25.4 ± 0.1 cm and 7.5 ± 0.2 cm. The result is 32.9 cm. To find the error in this result, consider that the first measurement lies between 25.3 and 25.5 cm. The second measurement lies between 7.3 and 7.7 cm. Therefore the sum lies between 32.6 and 33.2 cm, which may be written as 32.9 ± 0.3 cm. The absolute error in the result is the sum of the absolute errors of the two measurements. In symbolic form this may be written:

\[(a ± \delta a) + (b ± \delta b) = (a + b) ± (\delta a + \delta b)\]

To generalize:

if \[P = Q_1 + Q_2 + ... + Q_n\]

where \[Q_1, Q_2, ... Q_n\] are all measured quantities,

then \[\delta P = \delta Q_1 + \delta Q_2 + ... + \delta Q_n\]

Now consider the subtraction of the two measurements from the previous discussion. The result is 25.4 cm – 7.5 cm = 17.9 cm. The limits of the range of the result are obtained by calculating

\[(25.4 \text{ cm} + 0.1 \text{ cm}) – (7.5 \text{ cm} – 0.2 \text{ cm}) = 25.5 \text{ cm} – 7.3 \text{ cm} = 18.2 \text{ cm}\]

and

\[(25.4 \text{ cm} – 0.1 \text{ cm}) – (7.5 \text{ cm} + 0.2 \text{ cm}) = 25.3 \text{ cm} – 7.7 \text{ cm} = 17.6 \text{ cm}\]

Thus the result is expressed as 17.9 ± 0.3 cm. Thus when quantities are subtracted the absolute error in the result is the sum of the absolute errors in the quantities.

Combining this with the result for addition, if \(P\) is calculated from the addition and/or subtraction of a number of measured quantities \(Q_i\),

\[P = Q_1 + Q_2 - Q_3 - Q_4 + ... + Q_n\]

then \[\delta P = \delta Q_1 + \delta Q_2 + \delta Q_3 + \delta Q_4 + ... + \delta Q_n\]
WHEN QUANTITIES ARE ADDED OR SUBTRACTED THE MAXIMUM POSSIBLE ABSOLUTE ERROR IN THE RESULT IS THE SUM OF THE ABSOLUTE ERRORS IN THE QUANTITIES.

Multiplication
Consider a rectangle measured to have sides \(a \pm \delta a\) and \(b \pm \delta b\). The area \(A\) and its absolute error \(\delta A\) are calculated as follows:

\[
A \pm \delta A = (a \pm \delta a)(b \pm \delta b) = ab \pm (\delta a)b \pm a(\delta b) \pm (\delta a)(\delta b)
\]

Factoring the product \(ab\) yields:

\[
A \pm \delta A = ab \left[1 \pm \frac{\delta a}{a} \pm \frac{\delta b}{b} \pm \frac{(\delta a)(\delta b)}{ab}\right]
\]

Noting that \(ab = A\), and ignoring the second order term \((\delta a)(\delta b)/ab\) since it will be small compared to the other terms in the square brackets, yields:

\[
\delta A = A \left[\frac{\delta a}{a} + \frac{\delta b}{b}\right]
\]

i.e.

\[
\frac{\delta A}{A} = \frac{\delta a}{a} + \frac{\delta b}{b}
\]

In general, if \(P\) is the product of measured quantities \(Q_1, Q_2, \ldots, Q_n\), then the relative error in \(P\) is

\[
\frac{\delta P}{P} = \frac{\delta Q_1}{Q_1} + \frac{\delta Q_2}{Q_2} + \ldots + \frac{\delta Q_n}{Q_n}
\]

\[
\delta P = \left[\frac{\delta Q_1}{Q_1} + \frac{\delta Q_2}{Q_2} + \ldots + \frac{\delta Q_n}{Q_n}\right]P
\]

THE MAXIMUM POSSIBLE RELATIVE ERROR IN A PRODUCT IS THE SUM OF THE RELATIVE ERRORS IN THE FACTORS. Noting that relative and percentage errors are related by \(\%\) error \(= 100 \times \text{(relative error)}\), the maximum possible percentage error in a product is the sum of the percentage errors in the factors.

Division
Let \(P\) be the result of the division of \(Q\) by \(R\).

i.e.

\[
P = \frac{Q}{R}
\]

Then

\[
P \pm \delta P = \frac{Q \pm \delta Q}{R \pm \delta R} = \frac{Q}{R} \left[1 \pm \frac{\delta Q}{Q}\right] = \left(\frac{Q}{R}\right) \left[1 \pm \frac{\delta Q}{Q}\right]
\]

Applying the binomial expansion and ignoring 2nd order terms yields:
\[
P \pm \delta P = \left(\frac{Q}{R}\right) \left[1 \pm \frac{\delta Q}{Q} \pm \frac{\delta R}{R}\right] = P \left[1 \pm \frac{\delta Q}{Q} \pm \frac{\delta R}{R}\right]
\]

so

\[
\delta P = P \left[\frac{\delta Q}{Q} + \frac{\delta R}{R}\right]
\]

i.e.

\[
\frac{\delta P}{P} = \frac{\delta Q}{Q} + \frac{\delta R}{R}
\]

\[
\therefore \delta P = \left[\frac{\delta Q}{Q} + \frac{\delta R}{R}\right] P
\]

**THE MAXIMUM POSSIBLE RELATIVE ERROR IN A QUOTIENT IS THE SUM OF THE RELATIVE ERRORS IN THE DIVISOR AND DIVIDEND.**

**Powers and Roots**

Using the result obtained for multiplication and realizing that the expression \(Q^n\) means to multiply \(Q\) by itself \(n\) times, if

\[
P = Q^n
\]

then

\[
\frac{\delta P}{P} = n \frac{\delta Q}{Q}
\]

**THE MAXIMUM POSSIBLE RELATIVE ERROR IN RAISING A BASE VALUE TO AN EXPONENT IS THE PRODUCT OF THE EXPONENT AND THE RELATIVE ERROR IN THE BASE.**

For a more general example, consider the quantity \(N\) given by the following expression:

\[
N = \frac{AB^4}{(CD)^{1/2}}
\]

where \(A, B, C, D\) are all measured quantities. The maximum possible relative error in \(N\) is

\[
\frac{\delta N}{N} = \frac{\delta A}{A} + 4 \frac{\delta B}{B} + \frac{\delta C}{C} + \frac{1}{2} \frac{\delta D}{D}
\]

Note that the error in \(N\) is influenced more by errors made in measuring \(B\) than any of the other quantities. In designing and performing such an experiment great care should be taken to measure \(B\) with the highest degree of precision attainable.

**SUMMARY**

1. All measured quantities have associated with them an uncertainty which can be expressed as an absolute or relative error and which limits the number of significant figures of these quantities and any results calculated from them.

2. When uncertain quantities are added or subtracted the maximum possible absolute error of the result is determined by adding the absolute errors of the separate quantities.

3. When uncertain quantities are multiplied or divided the maximum possible relative error of the result is determined by adding the relative errors of the separate quantities.
4. When an uncertain quantity is raised to a power (including a fractional power) the maximum possible relative error in the result is determined by multiplying the relative error in the quantity by the exponent.

5. When uncertain quantities are related in more complicated ways (e.g. log functions, trigonometric functions) the result must be calculated twice, once with the experimental values and then with the extreme values of the quantities. The maximum possible absolute error is the difference between these two values of the complicated function.

These basic error propagation rules can be applied to elementary trigonometric functions as well, using the following sequence: **(Be sure to perform all three steps)**

**First**, derive the relative error equation in the usual way, treating the trigonometric function and its argument as a single variable:

i.e., if the equation is \( \nu_x = (\nu) (\cos \theta) \), the relative error equation would be

\[
\frac{\delta \nu_x}{\nu_x} = \frac{\delta \nu}{\nu} + \frac{\delta (\cos \theta)}{(\cos \theta)}
\]

Therefore

\[
\delta \nu_x = \left[ \frac{\delta \nu}{\nu} + \frac{\delta (\cos \theta)}{(\cos \theta)} \right] \nu_x
\]

**Second**, replace the relative error in the trigonometric function with its mathematical equivalent from the set given below:

\[
\delta (\cos \theta) = (\delta \theta)(\sin \theta), \quad \delta (\sin \theta) = (\delta \theta)(\cos \theta), \quad \delta (\tan \theta) = (\delta \theta)/(\cos \theta)^2
\]

(These are derived using differential calculus, and cannot be directly obtained using our basic rules. They also require the ERROR in the angle (not the angle itself) to be expressed in Radians)

Then, in our example,

\[
\delta \nu_x = \left[ \frac{\delta \nu}{\nu} + \frac{(\delta \theta)(\sin \theta)}{(\cos \theta)} \right] \nu_x
\]

Normally, an error equation in this form would suffice. However, the trigonometric function in the denominator could potentially equal zero, causing a division by zero.

**Third**, back-substitute the original equation into the error equation, expand and simplify:

i.e., since \( \nu_x = (\nu) (\cos \theta) \), replace \( \nu_x \) in the above equation with \( (\nu) (\cos \theta) \):

\[
\delta \nu_x = \left[ \frac{\delta \nu}{\nu} + \frac{(\delta \theta)(\sin \theta)}{(\cos \theta)} \right] \nu (\cos \theta) = \frac{\delta \nu}{\nu} \nu (\cos \theta) + \frac{(\delta \theta)(\sin \theta)}{(\cos \theta)} \nu (\cos \theta)
\]

which simplifies to

\[
\delta \nu_x = \delta \nu (\cos \theta) + (\delta \theta)(\sin \theta) \nu
\]

**NOTE**: For the units to work out correctly in this type of equation, \((\delta \theta)\) [and only \((\delta \theta)\)], **must** be expressed in **RADIANS**. \((\delta \theta^\circ \times 2\pi/360 = \delta \theta^{rd})\)
Examples

1. \[ P = (m + M)\nu \] (1)
   
   There is both addition and multiplication in this equation. Make the substitution
   \[ a = m + M \] (2)
   
   As this is addition only, we add absolute errors.
   \[ \delta a = \delta m + \delta M \] (3)
   
   Substituting equation (2) into equation (1) yields
   \[ P = a\nu \]
   
   which is multiplication only so we add relative errors:
   \[
   \frac{\delta P}{P} = \frac{\delta a}{a} + \frac{\delta \nu}{\nu} = \frac{\delta m + \delta M}{m + M} + \frac{\delta \nu}{\nu}, \text{ so } \delta P = \left[ \frac{\delta a}{a} + \frac{\delta \nu}{\nu} = \frac{\delta m + \delta M}{m + M} + \frac{\delta \nu}{\nu} \right] P
   \]

2. When resistances \( R_1, R_2, \) and \( R_3 \) are connected in parallel the total resistance, \( R \), is given by the equation
   \[
   \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
   \] (1)
   
   Make the substitutions \( a = \frac{1}{R}, b = \frac{1}{R_1}, c = \frac{1}{R_2}, d = \frac{1}{R_3} \)
   
   Equation (1) becomes \( a = b + c + d \) which is addition so we add absolute errors.
   \[ \delta a = \delta b + \delta c + \delta d \] (2)
   
   From \( a = \frac{1}{R} \), \( \delta a \) can be found. This is division so we add relative errors,
   \[
   \frac{\delta a}{a} = \frac{\delta 1}{1} + \frac{\delta R}{R} = 0 + \frac{\delta R}{R} = \frac{\delta R}{R}
   \]
   
   but absolute errors are needed to substitute into equation (2)
   \[
   \delta a = a \frac{\delta R}{R} = \left( \frac{1}{R} \right) \frac{\delta R}{R} = \frac{\delta R}{R^2}
   \]
   
   Similarly for \( \delta b, \delta c \) and \( \delta d \) we get
   \[
   \delta b = \frac{\delta R_1}{R_1^2}, \delta c = \frac{\delta R_2}{R_2^2}, \delta d = \frac{\delta R_3}{R_3^2}
   \]
   
   Substituting these values into equation (2):
   \[
   \frac{\delta R}{R^2} = \frac{\delta R_1}{R_1^2} + \frac{\delta R_2}{R_2^2} + \frac{\delta R_3}{R_3^2}
   \]
ERROR CALCULATION QUESTIONS

These questions are designed to help you in reviewing error calculations. In each of the questions you will be given an equation and some data to use in the equation. Each question consists of four parts.

(a) Give the relative error equation. The relative error equation should be in symbols only.

(b) Use the data to do the calculation. Watch your significant figures and the units.

(c) Calculate the relative error and the percentage error.

(d) Calculate the absolute error.

Sample

The equation of kinetic energy is \( KE = \frac{1}{2}m\nu^2 \).

\( m = 5.32 \pm 0.03 \text{ kg} \)
\( \nu = 2.37 \pm 0.07 \text{ m/s} \)

(a) \( \frac{\Delta KE}{KE} = \frac{\Delta m}{m} + 2 \frac{\Delta \nu}{\nu} \)

(b) \( KE = \frac{(5.32 \text{ kg})(2.37 \text{ m/s})^2}{2} = 14.9 \text{ J} \)

(c) \( \frac{\Delta KE}{KE} = \frac{0.03 \text{ kg}}{5.32 \text{ kg}} + 2 \frac{0.07 \text{ m/s}}{2.37 \text{ m/s}} = 0.065 = 6.5\% \)

(d) \( \Delta KE = 0.065 \times 14.9 \text{ J} = 1.0 \text{ J} \)
\( KE = 14.9 \pm 1.0 \text{ J} \)

1. The equation of an ellipse is

\[ y = b \sqrt{1 - \frac{x^2}{a^2}}, \]

\( a = 22.6 \pm 0.1 \text{ cm}, b = 46.9 \pm 0.1 \text{ cm}, x = 12.2 \pm 0.1 \text{ cm}: \ y = 39.48 \pm 0.29 \text{ cm} \)
2. From Kepler's third law of planetary motion, the orbital period $T$ of a satellite in a circular orbit at height $h$ above a massive body of radius $R$ and mass $M$ is:

$$T = 2\pi \sqrt{\frac{(R+h)^3}{GM}},$$

$R = (1.738 \pm 0.001) \times 10^6$ m; $h = (1.1 \pm 0.1) \times 10^6$ m

$G = 6.67 \times 10^{-11}$ N·m$^2$/kg$^2$; $M = (7.35 \pm 0.01) \times 10^{22}$ kg

$T = (1.357 \pm 0.073) \times 10^4$ s

3. Consider an Atwood's machine in which the pulley is neither frictionless nor massless. In that case the acceleration is

$$a = \frac{g(m_2 - m_1)}{(I/R^2) + m_1 + m_2 + m}$$

where $I$ is the moment of inertia of the pulley, $R$ is its radius and $m$ is a mass that is used to counteract the friction. Given $g = 980$ cm/s$^2$, $m_1 = 100 \pm 1$ g, $m_2 = 200 \pm 2$ g,

$I = 950 \pm 50$ g·cm$^2$, $R = 3.5 \pm 0.1$ cm and $m = 5.0 \pm 0.1$ g: $a = 256 \pm 15$ cm/s$^2$. 

H. SAMPLE CALCULATIONS

For each different equation used in the analysis of an experiment a sample calculation (and possibly a sample error calculation) are required. The following discussion explains what is required in a proper sample calculation. Steps 6, 7, and 8 may not be required – ask instructor.

1. Label the calculation descriptively. i.e. Explicitly state, **in words**, what is being calculated. If this is one of many similar calculations, specify which one.
2. State the values, with their units and experimental errors, that will be used in the calculation.
3. State the equation in symbols.
4. Show the substitution of values (with their units but not experimental errors) into the equation.
5. Give the result of the calculation, with correct units. (*Don’t round off*. Keep extra significant figures at this point, especially trailing zeros. Rounding will be done later.)
6. If required, state the error equation in symbols.
7. If required, show the substitution of values (with units) into the error equation.
8. If required, give the unrounded result of the error calculation, with units.
9. State the result and its experimental error with the appropriate number of significant figures.

In general, the following rule is to be used in rounding off the results of calculations: **ROUND OFF THE ABSOLUTE ERROR IN THE RESULT TO TWO SIGNIFICANT FIGURES, THEN ROUND OFF THE RESULT TO THE SAME NUMBER OF DECIMAL PLACES AS THE ABSOLUTE ERROR.** If scientific notation is being used, express both the result and its absolute error to the same power of ten before applying the second part of this rule.

As an example, suppose the magnitude of the momentum of a car on an air track whose mass and speed have been measured is to be calculated. The following is the correct sample calculation:

1. Sample Calculation of Momentum of Car on Air Track for Trial #1. (*label*)
2. \( m = 0.675 \pm 0.001 \text{ kg} \) *statement of values*
   \( \nu = 0.123 \pm 0.005 \text{ m/s} \)
3. \( p = m\nu \) *equation in symbols*
4. \( p = (0.675 \text{ kg})(0.123 \text{ m/s}) \) *substitution of values*
5. \( p = 0.083025 \text{ kg·m/s} \) *calculation result*
6. \( \delta p = \left( \frac{\delta m}{m} + \frac{\delta \nu}{\nu} \right) p \) *error equation in symbols*
   or \( \delta p = m \delta \nu + \nu \delta m \) *(if back-substitution was employed)*
7. \( \delta p = \left( \frac{0.001 \text{ kg}}{0.675 \text{ kg}} + \frac{0.005 \text{ m/s}}{0.123 \text{ m/s}} \right)0.083025 \text{ kg·m/s} \) *substitution of values*
8. \( \delta p = 0.003498 \text{ kg·m/s} = 0.0035 \text{ kg·m/s} \) *error calculation result*
9. \( p = (0.0830 \pm 0.0035) \text{ kg·m/s} \) or \( p = (8.30 \pm 0.35) \times 10^{-2} \text{ kg·m/s} \) *round off*
I: COMPARING TWO QUANTITIES

Experiments sometimes involve determining a physical quantity by two different methods to see if the results of the two methods agree. Sometimes one wishes to compare the experimental results with the predictions of a theory. In either case it is highly unlikely that there will be exact numerical agreement between the values to be compared. To compare the quantities, and decide if there is agreement, their experimental errors must be calculated. If the ranges of the two values overlap the values are said to agree within the limits of the error of that experiment. If the ranges do not overlap, then the values do not agree.

For example, if one determines the value of \( g \) to be \( 979 \pm 3 \) cm/s\(^2\) and the known value is \( 981 \) cm/s\(^2\) then the two values agree because the range of the experimental value (976 cm/s\(^2\) to 982 cm/s\(^2\)) includes the known value.

Bar Graph Comparison

\[
g_{\text{exp}} = 979 \pm 3 \text{ cm/s}^2
\]

Algebraic Comparison

The test for error range overlap can be expressed quantitatively as follows. Let \( x_1 \pm \Delta x_1 \) and \( x_2 \pm \Delta x_2 \) be two values that are to be tested for agreement within experimental error. If the magnitude of the difference between the values is less than the sum of their absolute errors then the values agree.

i.e. if \( |x_1 - x_2| \leq \Delta x_1 + \Delta x_2 \) is a true statement,

then \( x_1 \) and \( x_2 \) are said to be equal, “within experimental error”.

When experimental errors have not been computed, the experimental value is compared with the standard known value by computing the percentage difference:

\[
\% \text{ difference} = \left| \frac{\text{experimental value} - \text{known value}}{\text{known value}} \right| \times 100\%
\]

When a physical quantity does not have an accepted value, the following equation is used to calculate the percentage difference between values determined by two experimental methods:

\[
\% \text{ difference} = \left| \frac{\text{larger value} - \text{smaller value}}{\text{smaller value}} \right| \times 100\%
\]

When the percentage change in a value is to be calculated, the following equation is used:

\[
\% \text{ change} = \left| \frac{\text{final value} - \text{initial value}}{\text{initial value}} \right| \times 100\%
\]