Compton scattering is another phenomenon of electromagnetic radiation which cannot be explained in terms of the wave theory. In this experiment an attempt will be made to verify the Compton scattering formula, derived from the quantum theory of electromagnetic radiation, and as a consequence, the mass of the electron will be determined.

Theory:

In 1920, A.H. Compton investigated the scattering of monochromatic x-rays (electromagnetic radiation) from various materials. He observed that after the scattering the frequency (energy) of the x-rays had changed, and had always decreased. From the point of view of classical (wave) electromagnetic theory, this frequency shift cannot be explained since the frequency is a property of the incoming electromagnetic wave and cannot be altered by the change of direction implied by the scattering. If, on the other hand, the incoming radiation is thought of as a beam of photons (electromagnetic quanta) then the situation becomes that of photons of energy $E = h\nu$ scattering from free electrons in the target material. Energy-momentum conservation, applied to this situation, predicts that the scattered photons will have energy $E' = h\nu' < E$, in complete agreement with Compton’s experiments.

The frequency shift will depend on the angle of scattering, and can be calculated from kinematics. Consider an incoming photon of energy $h\nu$ and momentum $h\nu/c$ scattering from any electron of mass $m$. $p$ is the momentum of the electron after scattering and $h\nu', h\nu'/c$ are the energy and momentum of the scattered photon. For momentum conservation, the three vectors $h\nu/c, h\nu'/c$, and $p$ must lie in the same plane (see Figure 1).

![Figure 1: Schematic diagram of the Compton scattering effect](image)

From energy conservation,

$$h\nu + mc^2 = h\nu' + \sqrt{m^2c^4 + p^2c^2}$$

(1)

where $m$ is the rest mass of the electron.
From momentum conservation,

\[
\frac{hv}{c} = \frac{hv'}{c} \cos \theta + p \cos \phi
\]

and

\[
0 = \frac{hv'}{c} \sin \theta - p \sin \phi
\]

where \(\theta\) is the photon scattering angle and \(\phi\) is the electron recoil angle. Solving (2) and (3) for \(pc \cos \theta\) and \(pc \sin \theta\) respectively, and squaring,

\[
p^2 c^2 \cos^2 \phi = h^2 v^2 - 2h^2 vv' \cos \theta + h^2 v'^2 \cos^2 \theta
\]

\[
p^2 c^2 \sin^2 \phi = h^2 v'^2 \sin^2 \theta
\]

Adding these two equations and using \(\sin^2 + \cos^2 = 1\),

\[
p^2 c^2 = h^2 v^2 - 2h^2 vv' \cos \theta + h^2 v'^2
\]

Squaring equation (1) and solving for \(p^2 c^2\) yields

\[
p^2 c^2 = h^2 v^2 - 2h^2 vv' + 2hmc^2 (v - v') + h^2 v'^2
\]

Subtracting equations (6) and (7),

\[
\frac{v - v'}{vv'} = \frac{h}{mc^2} (1 - \cos \theta)
\]

Converting frequency to wavelength \((v = c/\lambda)\) gives

\[
\Delta \lambda = \lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)
\]

for the shift in wavelength of the scattered beam, while converting frequency to energy \((v = E/h)\) yields

\[
E' = \frac{E}{1 + \frac{E}{mc^2} (1 - \cos \theta)}
\]

for the energy of the scattered photons. Re-arranging equation (10) yields

\[
\frac{1}{E'} = \frac{1}{E} + \frac{1}{mc^2} (1 - \cos \theta)
\]

where \(E\) is the incoming photon energy and \(E'\) is the scattered photon energy. Thus the Compton formula, equation (11), predicts that the inverse of the energy of the scattered photon varies linearly as \((1 - \cos \theta)\) where \(\theta\) is the photon scattering angle.
Apparatus:

The source of photons (electromagnetic radiation) for this experiment is 50 mCi of $^{137}$Cs. $^{137}$Cs emits 0.662 MeV gamma rays. Although Compton’s experiments were done with x-rays, a gamma ray source should work just as well, since both x-rays and gamma rays are forms of electromagnetic radiation. The source is placed inside a lead collimator. DO NOT REMOVE THE SOURCE FROM THE LEAD COLLIMATOR. EXPOSURE TO THE RADIATION WILL BE HAZARDOUS. SPEND AS LITTLE TIME AS NECESSARY STANDING IN FRONT OF THE SOURCE OPENING.

The scattering target is a 1 cm diameter aluminum rod. The rod may be removed by unscrewing it from the baseplate.

The detection and analysis system consists of a NaI(Tl) scintillation crystal and photomultiplier tube, a scintillation amplifier/high voltage power supply, and a personal computer with a multi-channel analyser card. Gamma rays passing into the NaI(Tl) crystal cause flashes of light (scintillations) inside the crystal. These flashes of light release electrons from the photocathode of the photomultiplier tube (by the photoelectric effect). The high voltage applied to the photomultiplier tube causes the electrons to be channelled through the various stages of the tube, with amplification of the number of electrons occurring at each stage. The result is a pulse at the output of the photomultiplier tube, the voltage of the pulse being proportional to the energy of the gamma ray incident on the crystal (provided the gamma ray stops in the crystal). After linear amplification the voltage pulse is digitized by the ‘analog to digital converter’ (ADC) card in the computer. The monitor displays the number of pulses versus channel number. The channel number is directly proportional to the photomultiplier tube pulse voltage and hence to the deposited gamma ray energy. The monitor thus shows the energy distribution of the gamma rays being detected.

The scintillation crystal/photomultiplier tube assembly is mounted inside a heavy lead shield to reduce background radiation. The assembly, with shielding, is mounted on a trolley which can be rotated about a vertical axis centred on the target. The scattering angle can be measured on a protractor attached to the baseplate. Figures 2 and 3 are a diagram and a picture of the apparatus.

Figure 2: Schematic diagram of apparatus
Figure 3: Picture of apparatus

Procedure and Experiment:

NOTE: Refer to Figure 2 for the definition of the scattering angle, \( \theta \), and note that the labelling of the protractor corresponds to \( 90^\circ - \theta \). i.e. \( \theta = 90^\circ \) – protractor reading.

Before the actual Compton scattering experiment can be performed, two calibrations are required. First, the conversion equation between channel number and energy is determined; second, the protractor reading corresponding to \( \theta = 0^\circ \) is found.

1. Log-in to the computer (there is no password).
2. Turn on the UCS30 device.
3. Double-click on the UCS30/USX icon on the desktop.
4. **Click on the Mode menu and select ‘PHA (Amp In)’, the 2\(^{nd}\) item on the list.**
5. Click on the Settings menu and select Amp/HV/ADC. The high voltage should be set to 450. Click the button to turn it On. Check that the coarse and fine amplifier gains are 32 and 1.40 respectively, the conversion gain is 2048, and that the Amp In Polarity is positive. (If any of these settings are different than stated above, consult the lab instructor.)
6. Allow five minutes for the high voltage power supply to warm up.
7. The system is now ready for use.

Energy Calibration

A number of plastic disk sources of known gamma energies are supplied. Determining the peak channel numbers for these known energies allows calibration of the equipment.

<table>
<thead>
<tr>
<th>Source</th>
<th>Energy (KeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{57})Co</td>
<td>122</td>
</tr>
<tr>
<td>(^{137})Cs</td>
<td>662</td>
</tr>
<tr>
<td>(^{22})Na</td>
<td>511</td>
</tr>
</tbody>
</table>
To avoid contamination of the data due to radiation from the large source, be sure the plug is in place, remove the scattering target, and rotate the trolley away from the incident direction. The plastic sources can be taped over the opening in the detector shielding. *Do not put the tape on the label side of the disk sources!*

Begin data acquisition of the gamma ray spectrum for the $^{57}$Co disk source and allow to continue until there is a clearly identifiable peak on the monitor display.

Define an appropriate ROI that brackets the gamma ray peak and **record the peak channel number (Centroid) and FWHM** (half the FWHM is the error in the peak channel number).

**DO NOT** erase the $^{57}$Co spectrum – repeat the process of determining the peak channel number (centroid) and FWHM for the $^{137}$Cs and $^{22}$Na sources. You should now have three ROIs defining the three calibration peaks.

Calibration is done as follows:

- From the Settings menu select Energy Calibrate and 3 point.
- If prompted, enter the units as KeV.
- Put the cursor in the ROI of one of the peaks, enter the corresponding energy (in KeV), and click OK.
- Repeat for the other two peaks.

Once the calibration procedure has been completed, the horizontal axis of the display will read energy in KeV.

As a check of the linearity of the system’s response, plot by hand the energy calibration curve, i.e. plot gamma ray energy vs. peak channel number. The equation of this line allows manual conversion from channel number to energy.

**Protractor Calibration**

The protractor reading corresponding to $\theta = 0^\circ$ is that angle for which the maximum counting rate from the large source (with plug removed) is obtained. To determine this angle, proceed as follows:

1. Remove the plastic disk source from in front of the detector.
2. Remove the plug from the main $^{137}$Cs source.
3. Rotate the detector to a position that corresponds approximately to $\theta = 0^\circ$.
4. Record the detector angle.
5. Set the UCS 30 to acquire data for 5 seconds by clicking on the Clock button and setting the ‘Live Time’ to 5.
6. Record the total number of counts detected in the 5 second interval by defining a Region of Interest that covers the complete spectrum.
7. Alter the detector position by 1° and repeat steps 4 to 6 until the detector position corresponding to the maximum counting rate is found.

For any detector position, the scattering angle, $\theta$, is then given by the difference between the maximum counting rate angle and the protractor reading.
Compton Scattering Measurements

Put the aluminum scattering target in place. Acquire gamma ray energy data (i.e. the gamma energy spectra) for scattering angles of 10° to 90°, in 10° increments. For each angle determine the energy of the gamma ray peak. That is, measure the energy of the scattered gamma rays for scattering angles of $\theta = 10^\circ$, 20°, 30°, ..., 90°. (At each angle, acquire data until a well-defined peak is obtained, then define a ROI around the peak to obtain the energy of the peak and the FWHM.)

Note that due to lower probability of scattering at higher angles, it may be necessary to acquire data for ten or fifteen minutes until a definite peak is visible, whereas at small angles a few minutes is sufficient.

When you are sure you have collected all the required data (check with the lab instructor), turn off the equipment as follows:

- Go to the Settings menu and turn off the High Voltage.
- Close the UCS 30 program.
- Turn off the UCS 30 device.
- Log-out from the computer.

Analysis

Analysis Goals:

1. Plot a graph of $1/E'$ versus $(1 - \cos \theta)$.
   - Does your graph agree with the Compton scattering formula, equation (11)?

2. From your graph determine the mass of the electron and compare with the accepted value of 0.511 MeV/c².

3. From your graph, determine the incident gamma ray energy and compare with the accepted value of 0.662 MeV.