ELECTRON DIFFRACTION

In this experiment the existence of wave properties of particles is demonstrated through observation of electron diffraction. The wavelengths of electrons diffracting from a polycrystalline aluminum foil target are measured and compared with the theoretical de Broglie wavelengths.

Theory:

In 1924 Louis de Broglie proposed that matter possesses wave as well as particle characteristics. The de Broglie wavelength of a particle is given by \( \lambda = \frac{h}{p} \) where \( h \) is Planck’s constant and \( p \) is the particle momentum.

\[
p = \sqrt{2m_o T + \frac{T^2}{c^2}}
\]

where \( T \) is the particle’s kinetic energy and \( m_o \) is its rest mass. At low particle energies (i.e. non-relativistic speeds) the classical equation

\[
p = \sqrt{2mT}
\]

is sufficient.

If particles have wave properties, they should exhibit reflection, interference, diffraction, etc. Electron diffraction can occur from the ordered arrangement of atoms in a crystal. The ‘diffraction grating’ is formed by the parallel planes of crystal atoms and the ‘grating constant’ is the plane separation, \( d \). According to the Bragg formulation of diffraction, electron waves incident on the crystal planes at an angle \( \theta \) will be scattered at an equal angle \( \theta \). Figure 1 shows two electron waves being scattered from consecutive crystal planes.

There will be constructive interference between the two waves after scattering if the path length difference is an integral number of wavelengths.
This is called the Bragg condition. If the orientation of the crystal relative to the incident electron beam is changed continuously, electrons will be detected only for those angles satisfying equation (3). At all other angles destructive interference occurs and the electron waves ‘cancel’ each other. For a given crystal, there are numerous sets of parallel planes, each with a unique plane separation. For example, Figure 2 shows just three of the many possible sets of scattering planes.

\[ 2d \sin \theta = n \lambda \]  

(3)

The wavelength path difference, \( n \), between consecutive scattering planes (the order of interference in equation (3)) and the scattering plane orientation are described by the Miller indices \( h, k, \ell \). \( n \) is the greatest common divisor of \( h, k, \) and \( \ell \). The scattering plane orientation is the plane containing the three points found by moving along the three axes of the unit crystal cell distances of \( h/n \) units, \( k/n \) units, and \( \ell/n \) units respectively, where one ‘unit’ is the length of the respective edge of the unit cell. For aluminum, the unit cell is cubic so the lengths of all edges of the unit cell are equal. This length is given the symbol \( a \). \( a = 0.404 \) nm for aluminum. The distance between consecutive scattering planes is

\[ d = \frac{a}{\sqrt{\left(\frac{h}{n}\right)^2 + \left(\frac{k}{n}\right)^2 + \left(\frac{\ell}{n}\right)^2}} \]  

(4)

Substituting equation (4) into equation (3) yields

\[ \lambda = \frac{2a \sin \theta}{\sqrt{h^2 + k^2 + \ell^2}} \]  

(5)
From crystallography it is found that for a face-centred cubic structure such as aluminum, diffraction occurs if all Miller indices are odd or all are even. That is, the only allowed $h$, $k$, $\ell$ values are:

$$ h^2 + k^2 + \ell^2 $${
<table>
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<tr>
<th>$h$</th>
<th>$k$</th>
<th>$\ell$</th>
<th>$\sqrt{h^2 + k^2 + \ell^2}$</th>
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<td>2</td>
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<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>4.899</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5.196</td>
</tr>
<tr>
<td>etc.</td>
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</tbody>
</table>

The target consists of polycrystalline aluminum. Since there are such a large number of crystals, oriented randomly, in the path of the electron waves, all the possible constructive interference orientations will be present. Thus the diffraction pattern will appear as a set of concentric rings. Each ring corresponds to a set of Miller indices, with increasing ring radius corresponding to increasing $\sqrt{h^2 + k^2 + \ell^2}$ value.

**Apparatus:**

The diffraction experiment is performed with a commercial unit. It functions like an oscilloscope except that the electron beam is intercepted by a target located a distance $D$ from the screen. The very thin target consists of polycrystalline aluminum. The electron beam energy is altered by changing the high voltage applied to the tube. Figure 3 is a schematic diagram of the electron diffraction unit.

**Figure 3**

Note that the angle between the incident direction and a diffraction ring is $2\theta$ where $\theta$ is the angle between the incident electrons and the scattering planes in the target that are responsible for the diffraction ring.
Also

$$\tan 2\theta = \frac{d}{2D}$$  \hspace{1cm} (7)

where \(d\) is the ring diameter.

\[
\therefore \theta = \frac{1}{2} \arctan \left( \frac{d}{2D} \right)
\]  \hspace{1cm} (8)

Substituting into equation (5),

$$\lambda = \frac{2a \sin \left( \frac{1}{2} \arctan \left( \frac{d}{2D} \right) \right)}{\sqrt{h^2 + k^2 + l^2}}$$  \hspace{1cm} (9)

For small angles, \(\tan x \approx x\) and \(\sin x \approx x\),

$$\tan 2\theta \approx 2\theta \approx \frac{d}{2D} \Rightarrow \theta \approx \frac{d}{4D}$$

\[
\therefore \sin \theta \approx \theta \approx \frac{d}{4D}
\]

\[
\therefore \lambda \approx \frac{ad}{2D\sqrt{h^2 + k^2 + l^2}}
\]  \hspace{1cm} (10)

\(a = 0.404\) nm for aluminum, \(D = 18.06\) cm.

**Procedure and Experiment:**

1. Before turning on the diffraction unit turn down the intensity and high voltage controls (maximum counterclockwise). Check that the ammeter is on the 0–50 \(\mu\)A scale.
2. Turn on the power, and allow two minutes warm-up time.
3. Set the high voltage to 10 kV.
4. Increase the intensity to between 5 and 10 \(\mu\)A. DO NOT EXCEED 10 \(\mu\)A.
5. Adjust the focus for the minimum beam size. The horizontal and vertical controls permit the beam to be scanned across the target.
NOTE: The high voltage is not well-stabilised; it will need to be re-adjusted from time to time. Also, the beam current (intensity), focus, position, and high voltage controls all interact with one another.

6. Adjust the beam position so that a clear ring pattern is observed. Does the pattern qualitatively agree with that predicted in the theory?

Using vernier calipers, measure the diameters of as many rings as possible at a fixed high voltage of 10 kV.

7. Determine the Miller indices corresponding to each of the rings you have measured. Simply assigning Miller indices in order will likely yield incorrect results as some rings may overlap or be too faint to observe. (i.e. rings corresponding to some of the Miller indices values may not be observable.) The procedure to follow is:

a) Assign indices of 1,1,1 to the first ring (smallest diameter).

b) Calculate the electron wavelength using equation (9) or (10). (If you decide to use equation (10), you must justify your decision by showing an appropriate calculation.)

c) Since the high voltage (electron energy) was kept constant, the wavelength calculated in (b) applies to all the measured rings. Therefore, the correct Miller indices for a ring are those that yield a value for wavelength that is closest to the value calculated in (b). Thus, the wavelength value from (b) can be used to calculate an experimental value for \( \sqrt{h^2 + k^2 + l^2} \) for each ring. The Miller indices assigned to a particular ring would then be those that have a \( \sqrt{h^2 + k^2 + l^2} \) with which the experimental value agrees within error.

d) Show your calculations leading to the assignment of the Miller indices for each ring.

9. Tabulate ring diameter, Miller indices, and calculated wavelength (i.e. once the Miller indices have been assigned to each ring, apply equation (9) or (10) to each ring’s data to obtain a wavelength value.) The wavelength values obtained should all be equal within experimental error.

10. Measure the diameter of the smallest ring (1,1,1) for voltages of 10, 9, 8, 7, 6, and 5 kV.

11. Calculate the electron wavelength corresponding to each voltage using equation (9) or (10).

12. Assuming no energy losses in the diffraction tube, the electron kinetic energy, \( T \), is given by \( T = eV \), where \( V \) is the tube voltage. Calculate the electron momentum from its kinetic energy. Calculate the de Broglie wavelength. Is it necessary to use the relativistic equation for \( p \)?

13. Compare electron wavelengths measured by electron diffraction to those calculated by de Broglie’s equation.

14. Plot the measured electron diffraction wavelengths versus the inverse of the electron momentum. i.e. Plot \( \lambda_{\text{diff}} \) vs. \( 1/p \). Does the shape of this graph agree with de Broglie’s theory? Compare the slope of the graph with the theoretical value predicted by de Broglie’s equation.

References:

Harnwell and Livinggood, Experimental Atomic Physics, McGraw-Hill, 1961, Chpt. 5
Kittel, Introduction to Solid-State Physics, John Wiley & Sons, Inc. 1957