Polarization

**Introduction:** The polarization of a beam of light expresses the orientation of the electric field vector associated with the propagating electromagnetic wave. Because electromagnetic waves have a vector character, this direction is important, and the response of materials can vary greatly depending on the relative orientation of the electric field and the material axis.

**Physics:** One very convenient way of handling polarization in a beam a light is via the use of the so-called “Jones calculus”, or Jones matrix techniques. Please see the Appendix for a review of these methods.

**Experimental Procedure:** In this lab experiment you will use a laser-detector system set up on an optical rail. You will introduce various polarizing films and other objects into the beam path and observe the effect on the light transmitted. In this way you will be able to quantitatively verify some aspects of polarization theory as well as experiment with some useful applications of polarization.

**Equipment required:** HeNe laser (λ=632.8 nm)
- Optical Rail system
- Optical Power Meter (Thorlabs S120C or equivalent)
- Polaroid polarizing sheets mounted in rotary dial mounts (3)
- Quarter-Wave Plate (QWP)
- Half-Wave Plate (HWP)
- Calcite Crystal
- Selenite Crystal
- Faraday Effect System (TeachSPIN)
- MOKE system

**Experiments**

**The “Law of Malus”**

(a) Set up two Polaroid films in rotary mounts on the optical rail. Set both rotary mounts to 0 degrees. The first polaroid in a system is commonly referred to as THE “Polarizer” P because it picks out a polarization from an initially unpolarized beam. The second Polaroid film plays the role of the “Analyzer” A, since it can be used to determine the polarization direction of a polarized beam, as we will now see. Measure the laser beam power after it has passed through both P and A, with both set to 0 degrees. This is your reference beam power (we will refer to this as the beam “Intensity” I. Strictly speaking beam intensity refers to

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beam power per unit area, but in this case the beam cross-sectional area will not change so we don’t need to worry about this aspect.) Record this initial beam “intensity” (after traversing both P and A) as $I_0$.

(b) Now record the intensity $I$ with the analyzer set at each of the following angles in turn: 15°, 30°, 45°, 60°, 75°

(c) Sketch the output beam intensity $I$ versus the angle setting $\theta$ of the analyzer A. Does it show a $\cos^2(\theta)$ dependence as expected from Malus’ law? (i.e. does $I = I_0 \cos^2(\theta)$)

**Retarders- Quarter-Wave Plates (QWP) and Half-Wave Plates (HWP)**

(a) Set the angle of the Polarizer P to 0°. Then insert the Quarter-Wave Plate (QWP) with its angle set to 45° into the path of the beam, between Polarizer P and Analyzer A. The QWP will affect the beam by shifting one polarization component by 90° relative to the other polarization component.

(b) Using the Analyzer A, verify that the output intensity does not change as the analyzer is rotated through a full circle. This (partially) confirms that the beam emerging from the QWP is now circularly polarized

**Birefringence**

**Calcite- ordinary and extraordinary rays**

(a) Calcite (a crystalline form of Calcium Carbonate) has two different refractive indices one for “ordinary” and one for “extra-ordinary” rays ($n_o$ and $n_e$)

(b) Qualitatively verify the existence of two rays, using the calcite crystal provided

**Selenite- Conical Refraction**

Selenite is a crystalline form of gypsum (calcium sulfate CaSO$_4$.2H$_2$O—the stuff used in plaster of Paris and wallboard/”gyproc”). It was highly prized in ancient times and sometimes used as a window material by the Romans. It exhibits an interesting phenomenon: conical refraction, due to its peculiar birefringence.

Use the selenite crystal provide to observe conical refraction with the HeNe laser beam.

**Polarimetry**

(a) Set the angle of the Polarizer P to 0°. Samples with various concentrations of dextrose (a simple sugar molecule) are provided. You will place each of these samples in turn on the sample mount

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(b) Using a fixed Polarizer angle P of 0°. For each dextrose solution sample in turn, find the angular setting of Analyzer A where the output intensity is a minimum.

(c) Subtract 90 degrees from each of the above angles and plot the results versus concentration. Fit a straight line to the data. The fact that these data fall on a straight line is the basis for polarimetry, a widely used method to determine sugar concentrations in unknown solutions.

Rotation of the Plane of Polarization A: Polarimetry

(prefatory note: it is important for the solution polarimetry experiment below that the dextrose solutions be well-mixed. Please ensure that all the dextrose is dissolved (i.e. no precipitate at the bottom of the flask) prior to the experiments.

(a) Set the angle of the Polarizer P to 0°. Samples with various concentrations of dextrose (a simple sugar molecule) are provided. You will place each of these samples in turn on the sample mount.

(b) Using a fixed Polarizer angle P of 0°. For each dextrose solution sample in turn, find the angular setting of Analyzer A where the output intensity is a minimum.

(c) Subtract 90 degrees from each of the above angles and plot the results versus concentration. Fit a straight line to the data. The fact that these data fall on a straight line is the basis for polarimetry, a widely used method to determine sugar concentrations in unknown solutions.

Rotation of the Plane of Polarization B: The Faraday Effect

(a) Set up the Faraday effect system (from TeachSPIN)

Rotation of the Plane of Polarization C: Magneto-Optic Kerr Effect (MOKE)

(a) Set up the MOKE system

Appendix: The Jones Calculus

Note: This discussion follows closely that of G.R. Fowles and E.Hecht in their respective textbooks [1], [2]. The book by the Pedrottis is also useful [3].

We’ve seen how polarized light can be represented by complex components

$$\vec{E}_0 = E_{0x} e^{i\phi_x} \hat{x} + E_{0y} e^{i\phi_y} \hat{y}$$

where we have allowed for the possibility of complex phase shifts on the x and y components of the electric field. We can write the electric field for a general polarized monochromatic light beam propagating along the z axis as:

$$\vec{E} = E_{0x} e^{i(kz - \omega t + \phi_x)} \hat{x} + E_{0y} e^{i(kz - \omega t + \phi_y)} \hat{y}$$

we can then write this as:

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\[ \vec{E} = [E_{0,x}e^{i\phi_x}\hat{x} + E_{0,y}e^{i\phi_y}\hat{y}]e^{i(kz-\omega t)} \]

where we have separated out the common wave propagation part \( e^{i(kz-\omega t)} \).

We can write the prefactor which contains the x and y electric field amplitudes \textit{as well as their relative phases} \( \phi_x \) and \( \phi_y \) as follows:

\[ \vec{E}_0 = \begin{bmatrix} E_{0,x}e^{i\phi_x} \\ E_{0,y}e^{i\phi_y} \end{bmatrix} \]

This is the Jones vector for fully polarized light. It is capable of describing any linearly or circularly or elliptically polarized light beam. The tilde symbol (~) over the electric field indicates that this is a Jones vector description.

**Normalization:** Note that this general Jones vector has magnitude \( \sqrt{E_{0,x}^2 + E_{0,y}^2} = |\vec{E}| \). It is convenient to tabulate the various possible Jones vectors for different polarization states (e.g. x or y linearly polarized, circularly polarized, and elliptically polarized states) in terms of normalized Jones vectors which have magnitude = 1. A general normalized Jones vector would then have the form:

\[ \vec{J} = \begin{bmatrix} a \\ b \end{bmatrix} \]

where \( a \) and \( b \) are in general complex with \( \sqrt{|a|^2 + |b|^2} = 1 \). Any desired Jones vector can then be obtained by multiplying the appropriate normalized Jones vector (see Table 1 below) by the desired electric field amplitude \( |\vec{E}| \).

Jones Matrices: The power of the “Jones Calculus” approach to polarized light optics lies in the fact that the effect of any polarization discriminating component (e.g. polarizers, quarter and half wave plates, etc.) can be modeled by multiplying the Jones vector for the input light beam by the Jones matrix for the polarization sensitive element. The result will be a Jones vector describing the final output polarization state. The derivation of the Jones matrices for different polarization sensitive elements will be discussed in the classroom lectures, but the end results for some useful and common polarization-sensitive elements are given in Table 2 below.

\[ \vec{E}_{out} = M\vec{E}_{in} \]

If we have a cascaded series of \( N \) polarization sensitive elements then the total Jones matrix for the system is the matrix product of the Jones matrices of the individual elements, as follows (note we have labeled the \( N \) elements with index \( i = 1,2,3 \ldots N \)):

\[ M_{TOT} = M_N M_{N-1} \cdots M_i \cdots M_1 \]
**Table 1: Normalized Jones Vectors**

Horizontal (x) Polarization: \( \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \)
Vertical (y) Polarization: \( \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \)
General: \( \left( \begin{array}{c} \cos \alpha \\ \sin \alpha \end{array} \right) \)

Left-Hand Circ. Pol. (LHCP): \( \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ i \end{array} \right) \)
Right-Hand Circ. Pol. (RHCP): \( \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ -i \end{array} \right) \)

**Table 2: Jones Matrices for Common Polarization-Sensitive Elements**

Horizontal \( \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \)
Vertical \( \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \)
45° \( \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \)

Quarter-Wave Plate QWP (top sign--slow axis vertical, bottom sign-- slow axis horizontal) \( e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & \pm i \end{bmatrix} \)

Half-Wave Plate (HWP) (top sign-- slow axis vertical, bottom sign-- slow axis horizontal) \( e^{i\pi/2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \)

Polarization Rotator \( (\alpha \rightarrow \alpha + \beta) \) : \[
\begin{bmatrix} 
\cos \beta & -\sin \beta \\
\sin \beta & \cos \beta 
\end{bmatrix}
\]

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References


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