Application of Monte Carlo Methods in Radiotherapy Physics

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• General Description of the Monte Carlo Method
  • Coupled Photon/Electron Transport
  • Modelling a Medical Linear Accelerator
    • Radiotherapy Dose Calculations
    • Lung Dosimetry
  • Small Field Dosimetry
Let’s start with a classic example

Area of square: $A_s = 1$
Area of circle: $A_c = \pi$
Fraction $p$ of random points inside circle:

$$p = \frac{A_c}{A_s} = \frac{\pi}{4}$$

Random points: $N$
Random points inside circle: $N_c$

$$\Rightarrow \quad \pi = \frac{4N_c}{N}$$
Simulation results... convergence
Define the Monte Carlo Method as follows:

Generate $N$ random "points" $\bar{x}_i$ in the problem space.

Calculate the "score" $f_i = f(\bar{x}_i)$ for the $N$ "points".

Calculate

$$
\langle f \rangle = \frac{1}{N} \sum_{i=1}^{N} f_i,
\quad
\langle f^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} f_i^2
$$

According to the Central Limit Theorem, for large $N$ $\langle f \rangle$ will approach the true value $\bar{f}$. More precisely,

$$
p(\langle f \rangle) = \frac{\exp \left[-(\langle f \rangle - \bar{f})^2/2\sigma^2\right]}{\sqrt{2\pi\sigma}},
\quad
\sigma^2 = \frac{\langle f^2 \rangle - \langle f \rangle^2}{N - 1}
$$

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Essential ingredients for MC particle transport simulations

- A random number generator
- Physics: interaction cross sections
- Transport: define the rules and geometry
- Bookkeeping: scoring the results of interest

• Photon and electron interactions with atoms and molecules are described by QED.

• Interactions are very simple within the energy range common to external beam radiation therapy.

- Let’s assume we have a well established random number generator
- We know how to sample from a probability density function
Photon Interactions

- Compton scatter (Incoherent)
  - Klein-Nishina equation

- Photo-electric absorption
  - Low keV energy range

- Pair Production
  - Minimum energy requirement

- Rayleigh scatter (Coherent)
  - Very small contribution at keV energies
Electron and positron interactions

- Inelastic collisions: with atomic electrons that lead to ionizations and excitations
  - Energy transfer large compared to binding energies

- Bremsstrahlung in the nuclear fields
  - Energy loss through photon emission due to acceleration

- Elastic collisions: with nuclei and atomic electrons
  - Many interactions, each with little energy transfer

- Positron annihilation with atomic electrons
  - Photon production

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(a) [Diagram showing electron and positron interactions]

(b) [Diagram showing energy losses and scattering]
Condensed History Steps and Variance Reduction Methods

- Range Rejection
- Photon Forcing
- Bremsstrahlung Splitting
- Russian Roulette

Condensed history steps

- A CH simulation only provides the positions $x_i$ and directions $\Omega_i$ of the particles at the beginning of the $i$'th step.
- No information is available on how the particle traveled from $x_i$ to $x_{i+1}$.
- Attempts to simply score e.g. energy at the positions $x_i$ can result in artifacts, unless the step-lengths are randomized.
- Attempts to simply connect $x_i$ with $x_{i+1}$ can result in artifacts unless the steps are short enough.

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- Condensed history method (Berger) uses multiple scattering theory (Goudsmit and Saunderson).
- Must contain a method for boundary crossing.
- PRESTA-II uses single scattering mode below an electron energy cut off (ECUT).
The EGSnrc system is a package for the Monte Carlo (MC) simulation of coupled electron-photon transport.

EGSnrc is an extended and improved version of the EGS4 package originally developed at SLAC.

Its current energy range of applicability is considered to be 1keV - 10 GeV.

It incorporates many improvements in the implementation of the condensed history technique for the simulation of charged particle transport and better low energy cross sections.
Modelling a Medical Linear Accelerator
What's inside a medical linear accelerator?
What do we need to model?
What do we have to work with?

**BEAMnrc User Code**

A component module can be considered as a block with a 'front' and 'back' surface.

An accelerator is built with many such blocks.

The **BEAMnrc convention** is to define the beam central axis as the z-axis with the source at z = 0.
How do we model each component in the linac head?

Source
- Incident electron beam is modelled as a monoenergetic circular 2-D Gaussian.

Components
- Simply define “zthick” layers and xy-coordinates.
- The geometry can be complex.
- Once it’s done... it’s done!
The end result is a phase space file

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Type of variable returned</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Position in X direction in cm</td>
<td>Real*4</td>
</tr>
<tr>
<td>Y</td>
<td>Position in Y direction in cm</td>
<td>Real*4</td>
</tr>
<tr>
<td>Z</td>
<td>Position in Z direction in cm</td>
<td>Real*4</td>
</tr>
<tr>
<td>U</td>
<td>Direction cosine along X</td>
<td>Real*4</td>
</tr>
<tr>
<td>V</td>
<td>Direction cosine along Y</td>
<td>Real*4</td>
</tr>
<tr>
<td>E</td>
<td>Kinetic energy in MeV</td>
<td>Real*4</td>
</tr>
<tr>
<td>Statistical_weight</td>
<td>Particle statistical weight</td>
<td>Real*4</td>
</tr>
<tr>
<td>Particle_type</td>
<td>Type of the particle</td>
<td>Integer*2</td>
</tr>
<tr>
<td>Sign_of_W</td>
<td>Sign of W (direction cosine in Z)</td>
<td>Logical*1</td>
</tr>
<tr>
<td>Is_new_history</td>
<td>Signifies if particle belongs to new history</td>
<td>Logical*1</td>
</tr>
<tr>
<td>Integer_extra</td>
<td>Extra storage space for variables (e.g., EGS LATCH, incremental history number, PENEOPE ILB, etc.)</td>
<td>n*(Integer*4) (n &gt; 0)</td>
</tr>
<tr>
<td>Float_extra</td>
<td>Extra storage space for variables (e.g., EGS ZLAST)</td>
<td>m*(Real*4) (m &gt; 0)</td>
</tr>
</tbody>
</table>
Sampling from a source instead of modelling the entire linac

A Three-Source Model for Clinical Photon Beams

- Point/extended source for primary photons
- Extrafocal source for scattered photons
- Extended source for contaminant electrons

C-M Charlie Ma et al AAPM Summer School 2006
Dose calculations in a voxelated geometry

- DOSXYZnrc is a general-purpose EGSnrc user-code for 3-D absorbed dose calculations.
- Scores energy deposition in a voxelated geometry.
- Voxel dimension, material and density are completely variable.
- Dose is calculated as a function of the initial electron histories incident on the Bremsstrahlung target.
Benchmarking a linac model

- 6.2 MeV mono-energetic electron beam with a 2-D Gaussian FWHM = 0.140 cm
- $2.5 \times 10^6$ initial histories
- Selective Bremsstrahlung splitting with a maximum splitting number = 1000
- Russian Roulette was turned off
Radiotherapy Dose Calculations
Head-and-neck IMRT treatment plan
Using DOSXYZnrc with CT data

- We want to do Monte Carlo dose calculations on patient specific CT data sets.
- Need a way of converting CT numbers to voxelated physical densities.
7-field IMRT

BEAMnrc field specific simulations
IMRT transport through the MLC
DOSXYZnrc simulation in the CT data set
Lung Dosimetry

• Fundamental lung dosimetry: respiration cycle and anatomy
PhD Progress Report
The effects of lung structure and function on the accuracy of dose calculations
Kerry Babcock M.Sc.
February 2, 2009

Dose Inaccuracy: Imaging
- CT representation
- Resolution ~ 1mm x 1mm x 2 mm
  - Lose small branches
  - Lose alveolar tissues
- Atomic composition and density averaged

Goal 1: To determine whether the inclusion/exclusion of lung structure below CT imaging resolution results in deviations in calculated dose distributions
Model of the Lung: Tissue

- Alveolar tissue
- Polyhedral “foam”
- 200 microns
- Approximate as dodecahedron shell
- Tissue wall/Air cavity

Model of the Lung: Branches

- "Breathing" outer boundary
- Bronchial/Arterial/Venous
Transport through the lung model has been verified and the initial results are being analyzed.

- Appears that lung tissue can be approximated as a uniform material.

- Branching structure may result in significant dosimetric differences.

- Code Models into MC

  - DOSXYZnrc → voxeled transport/scoring of dose
  - Re-wrote DOSXYZnrc to transport independent of voxel boundaries
  - Benchmark modified code for several phantoms
  - My Published Results:
    - Med Phys 35 Increasing the speed of DOSXYZnrc Monte Carlo simulations through the introduction of nonvoxelated geometries
Small Field Dosimetry

• Protocol like application of detector specific correction factors

• Modelling various detectors for small field reference dosimetry
Stereotactic Radiotherapy
Application of MC to small field dosimetry

For standard radiotherapy dosimetry ionization chamber readings are made under charged particle equilibrium.

\[ Q = Q_{\text{raw}} \cdot P_{\text{ion}} \cdot P_{\text{elec}} \cdot P_{\text{TP}} \cdot P_{\text{pol}} \]

Alfonso et al. define a field factor that converts absorbed dose to water between the machine specific reference field and the clinical field of interest.

\[ D_{w, Q_{\text{clin}}} = D_{w, Q_{\text{msr}}} \cdot \Omega_{Q_{\text{clin}}, Q_{\text{msr}}} \]

The field factor can be measured as the ratio of corrected detector readings \( M \) multiplied by a Monte Carlo calculated correction factor.

\[ \Omega_{Q_{\text{clin}}, Q_{\text{msr}}} = \frac{M_{Q_{\text{clin}}}}{M_{Q_{\text{msr}}} \cdot \left( \frac{D_{w, Q_{\text{clin}}}}{D_{w, Q_{\text{msr}}}} \cdot \frac{D_{f_{\text{clin}}, Q_{\text{clin}}}}{D_{f_{\text{msr}}, Q_{\text{msr}}}} \right)} \]

**Detector Selection**

Wellhofer/Scanditronix

- **IC15** → Active Volume = 0.13 cm³  
  Inner Diameter = 6.0 mm  
  Electrode Diameter = 1.0 mm

- **IC04** → Active Volume = 0.04 cm³  
  Inner Diameter = 4.0 mm  
  Electrode Diameter = 1.0 mm

- **IC01** → Active Volume = 0.01 cm³  
  Inner Diameter = 2.0 mm  
  Electrode Diameter = 0.35 mm

**Stereotactic Field Diode (SFD)**

- *p*-type silicon diode

  - Chip Size = 0.95 mm²
  - Chip Thickness = 0.5 mm
An EGSnrc Monte Carlo study of the SFD Si-diode for small field relative dosimetry

“... a work in progress”

Gavin Cranmer-Sargison§ and Kerry Babcock‡

§Saskatoon Cancer Centre, Saskatchewan Cancer Agency
‡Department of Physics, University of Saskatchewan
PEGS4 Data

Defined as an element, compound or mixture.

ABS ($\rho = 1.04$ g·cm$^{-3}$)
Compound $C_6H_8 \cdot C_4H_6 \cdot C_3H_3N$

Epoxy ($\rho = 1.02$ g·cm$^{-3}$)
Compound generated from existing data

Coaxial cable ($\rho = 3.33$ g·cm$^{-3}$)
Mixture of copper and polyethylene

Using the same geometry, we also modelled a silicon chip in water and all water.
EGSnrc Parameters and Experimental Set-up

- ECUT = 0.521 MeV
- PCUT = 0.001 MeV
- ESTEP = 0.25
- Electron step algorithm = PRESTA-II
- Boundary crossing algorithm = EXACT
- Electron range rejection ESAVEIN = 2 MeV
- IFULL = entrance regions
- History Number = $8.0 \times 10^9$
- NRCYL was calculated and set automatically

- All simulations and measurements were made for an isocentric set-up
- Measurement depths at 1.5, 5.0 and 10.0 cm
- Three independent experimental sessions
- Mean ROF was calculated from five readings
- Jaw collimated 5.0 cm square field as our machine-specific field (msr)
- Clinical field sizes (clin) of 4.0, 3.0, 2.0, 1.0, 0.9, 0.8, 0.7, 0.6 and 0.5 cm.
## Output Factors
### Simulation and Measurement

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<thead>
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<th>Field Size</th>
<th>Depth = 1.5 cm</th>
<th>Meas. (1 SD)</th>
<th>Depth = 5.0 cm</th>
<th>Meas. (1 SD)</th>
<th>Depth = 10.0 cm</th>
<th>Meas. (1 SD)</th>
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</thead>
<tbody>
<tr>
<td>5.0</td>
<td>1.000 (0.015)</td>
<td>1.000 (0.000)</td>
<td>1.000 (0.014)</td>
<td>1.000 (0.014)</td>
<td>1.000 (0.014)</td>
<td>1.000 (0.000)</td>
</tr>
<tr>
<td>4.0</td>
<td>0.970 (0.014)</td>
<td>0.981 (0.001)</td>
<td>0.984 (0.013)</td>
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<tr>
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<td>0.958 (0.000)</td>
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<tr>
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<tr>
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<td>0.803 (0.013)</td>
<td>0.794 (0.001)</td>
<td>0.769 (0.013)</td>
<td>0.765 (0.001)</td>
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<tr>
<td>0.9</td>
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<td>0.825 (0.003)</td>
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<td>0.615 (0.014)</td>
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<tr>
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<td>0.607 (0.013)</td>
<td>0.565 (0.015)</td>
<td>0.549 (0.002)</td>
<td>0.535 (0.014)</td>
<td>0.525 (0.005)</td>
</tr>
</tbody>
</table>
Results as of this past weekend

- Find a silicon diode substrate commonly used in other detectors that would produce results consistent with experiment.

- We have identified 6H-SiC as a potential next choice for the SFD model.

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<th>Water (1 SD)</th>
<th>Meas. (1 SD)</th>
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</tr>
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Engineering Physics 495: Capstone Design Project

**Linear Actuator and Hardware**

- Stepper motor: UltraMotion, Cutchoque, NY
- 10 µm resolution per motor step

**Microcontroller**

- Arduino Diecimila Microcontroller board

**User Interface**

- Displays current position
- Input displacement
- Display position and electrometer reading
- Position and measurement logging
- Internal operations
  - Input and output error checking
  - Output to microcontroller

P. Bazylewski, K. Kreuger and E. Normand
Department of Physics and Engineering Physics, University of Saskatchewan
• Benchmark our linac model for small field dosimetry
  • Model the small volume ion chambers
  • May need imaging to model the detectors correctly

• Experimental measurements using the linear actuator
  • Effect of positional uncertainty on ROFs
  • Test the limits of the commercial water tank for small field

... is only weeks away!
Thank-You!