**Introduction:** Interferometers are the key to accurate distance measurement using optics. Historically, when mechanical measurements dominated, interferometers provide the first accurate measurements of the wavelength of light. Today, when optical wavelengths are known extremely accurately, interferometric techniques allow precise determination of mechanical distances in terms of precisely known wavelengths of light. Many different types of interferometers exist; all rely on interference effects involving two or more beams of light, but they differ greatly in their configuration and uses. In this lab we will explore a variety of interferometer configurations with different applications.

**Equipment required:** Na lamps
- Michelson Interferometer
- Fabry-Perot Interferometer
- Shear Interferometer (to test beam collimation)
- Holographic Interferometer (Digital) using HeNe laser ($\lambda=632.8$ nm)

**PART 1: Michelson Interferometer**
The Michelson interferometer is one of the most useful of all optical instru-

![Michelson Interferometer](image)

*Figure 1: Michelson Interferometer*

ments. It was originally designed by Michelson and Morley to detect the "ether" medium in which light waves were supposed to propagate, just as sound waves propagate in air. The negative result of that experiment led Einstein to postulate the special theory of relativity based on the principle that the speed of light is the same in all inertial reference frames. These days, the Michelson interferometer is used for very accurate determinations of the wavelength of spectral lines. In fact, until recently, the meter was defined as 1,650,763.73 times the wavelength of the orange-red spectral line of Krypton-86, and the Michelson interferometer was one of the instruments used by the National Bureau of Standards to measure that wavelength accurately.

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The apparatus basically consists of a half-silvered beam-splitting mirror M3 from which half of the light travels to mirror M1 and is reflected, while the other half of the light goes to mirror M2 and is reflected.

![Figure 2: Michelson Interferometer with compensating plate]

1. Michelson: Interference when light of a single wavelength is used:

Suppose the light source produces light waves of a given wavelength $\lambda$. These incident waves are incident on the beam splitter, and can be written as

$$E_0 = A \sin (kx - \omega t - a)$$

$$= A \sin (2\pi x/\lambda - 2\pi f t - a)$$

where $k = 2\pi/\lambda$ is the propagation constant (propagation assumed to be in air so the refractive index =1), and $\omega$ is the angular frequency.

If we set

$$f(t) = \omega t - a$$

$$E_0 = A \sin (kx - f(t))$$

Let us define the origin $x=0$ to be at the position of the beam-splitting mirror $x = 0$ (we can do this without loss of generality)

$$E_0 = A \sin (-f(t))$$

When the incident light encounters the beamsplitter mounted at $45^\circ$ (assumed ideal for now), half the beam is reflected and as a consequence its path changes by $90^\circ$ and it travels a distance $l_1$ to fully reflecting mirror M1, where it is reflected, reverse direction, and returns back to the beamsplitter, having traveled a total distance $2l_1$ (for the moment we are ignoring the compensating plate). Meanwhile the transmitted half of the beam has likewise travelled a distance $l_2$ to fully reflecting mirror M2, undergone a reflection, and reversed course, travelling a total distance $2l_2$ to arrive back at the beamsplitter location. At the beamsplitter the electric field amplitudes of the two returned light waves are then:

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E_1 = a \sin (2k l_1 - f(t) - \pi)
E_2 = a \sin (2k l_2 - f(t))

The difference of phase of \pi between the two returned waves arises because half of the incident beam reflects externally from the beam-splitting mirror (after travelling to M1) while the other half reflects internally at the beam-splitting mirror (after travelling to M2). In the first case, the beam is travelling in air, and reflecting at the air/glass interface; in the second case, the beam is travelling in glass and reflecting at the glass/air interface. (A little consideration of the Fresnel equations for the phase shifts experienced by the light waves upon reflection will hopefully make this clear.) This difference in reflection boundary conditions experienced by the two beams is what leads to the net phase difference of \pi.

With our eye (or another photodetector) we view the intensity (or irradiance) associated with the total electric field E_{tot} = (E_1 + E_2). As we saw in class the intensity of the light beam is given by \( I = \frac{E_{tot}^2}{2Z_0} \), where \( Z_0 \) is the characteristic impedance of free space. So first we need to add the electric field amplitudes, and then square the result. Using the trigonometric identity:

\[
\sin a + \sin b = 2 \cos \left(\frac{a-b}{2}\right) \sin \left(\frac{a+b}{2}\right)
\]

we obtain

\[
E_{tot} = E_1 + E_2 = 2a \cos \left[ k(l_1 - l_2) - \pi/2 \right] \sin \left[ k(l_1 + l_2) - \pi/2 f(t) \right]
\]

The eye detects the intensity of the wave, which as we have seen is proportional to the time average of the square of the electric field \( E_{tot} \):

\[
I \sim E^2 = 4a^2 \cos^2 \left[ k(l_1 - l_2) - \pi/2 \right] \sin \left[ k(l_1 + l_2) - \pi/2 f(t) \right]
\]

In the time average, only the last term on the right enters, and since the time average of \( \sin^2 \) is \( \frac{1}{2} \)

\[
I \sim E^2 = 4a^2 \cos^2 \left[ k(l_1 - l_2) - \pi/2 \right] \frac{1}{2} = 2a^2 \sin^2 \left[ k(l_1 - l_2) \right]
\]

where we have also used the identity \( \cos(\theta-\pi/2) = \sin \theta \)

The maxima of observed optical intensity \( I \) thus occur when \( \sin[k(l_1 - l_2)] = \pm 1 \). Since \( k = 2\pi/\lambda \):

Maxima of intensity occur when \( (l_1 - l_2) = \lambda / 4, 3 \lambda / 4, 5 \lambda / 4, \ldots \)

This is shown in Fig. 3 below:
We see that movement of M1 by $\lambda/2$ causes one complete interference fringe to pass by (i.e. the observed intensity goes from a maximum to a minimum and then back to a maximum again). Thus, by counting the number of fringes that pass by when the micrometer screw changes the position of M1 by a given amount, we can determine the wavelength of the light used.

By determining the mirror movement between the individual fringes, the average wavelength can be calculated.

**Michelson: Procedure:** With reference to the Michelson configuration in Fig. 2: the reflected light beams from the two mirrors then recombine at M3 and are examined by eye as shown. Whether the interference between the two beams will be constructive or destructive depends upon the path lengths in the two arms. Notice that movement of mirror M1 by one-half wavelength will cause the beams to undergo a net path difference of one whole wavelength. The purpose of the compensating plate is to ensure that both beams travel through equal path lengths in glass. The compensating plate is exactly equal in thickness to mirror M3. In the diagram shown, you can see that each beam passes through 3 thicknesses of glass in going from the source to your eye.

Mirror M2 has two tilt adjustment screws which can be used to align M2 with mirror M1 mounted on the carriage. The carriage is movable by means of a micrometer screw which actuates a pivoted beam coupled to the carriage. **The beam provides a 5:1 reduction from the indicated micrometer reading to the actual length traversed by the carriage.** The micrometer itself has 25 mm of movement and vernier graduations for reading to 0.01 mm, hence the carriage has 5 mm movement which can be read to 0.002 mm.
As shown in Fig. 3 above, the movement of mirror M1 by a distance $\lambda/2$ causes one complete interference fringe to pass by (i.e. from bright to dark to bright again). Count the number of fringes that pass by when the micrometer screw changes the position of mirror M1 by a given amount (you will need to record the initial and final micrometer settings for the screw position). Use this measurement to determine the wavelength of the light used. Note that because the yellow light from the Na lamp is a “doublet” and actually consists of two very close but different atomic emission wavelengths, what you are actually measuring this way is the average wavelength.

The fact that the sodium lamp produces two closely spaced wavelengths (a doublet) results in a variation in fringe visibility as moveable mirror M1 is moved over larger distances. This variation can easily be observed. It is described in more detail in the Appendix.

We will use the Fabry-Perot interferometer in the next Part to measure the doublet separation.

**Part 2: Fabry-Perot Interferometer**

The Fabry-Perot configuration (see Fig. 4) consists of two partially-reflecting mirrors separated by a distance $L$. This widely used instrument was first constructed in the early 1800s by Charles Fabry and Alfred Perot. The Fabry-Perot interferometer has an extremely high resolving power - about 10 times better than a grating spectrometer (which is already at least an order of magnitude better than a prism spectrometer). As such it has many applications in precision measurement, and is often referred to as an “etalon” (the French word for “stallion”, which has come to mean “a standard of measurement”, for reasons that are not entirely clear). The Fabry-Perot etalon configuration is widely used in precision spectroscopy, precision distance measurement, and it also serves as the optical “resonator” cavity for most lasers. Thus, it is worth studying in some detail.

![Fabry-Perot optical resonator and the Fabry-Perot interferometer (schematic)](image)


**Figure 4: Fabry-Perot Etalon**

Fig. 4 shows a schematic diagram of a Fabry-Perot etalon configuration. The complete interferometer consists of a Fabry-Perot etalon, and a lens system or eyepiece to focus the light either onto a screen or,
for the instrument you will use in this lab, at the observation point (your eye will be effectively at the position of the screen in the diagram).

When a broad monochromatic light source is used as the input to the interferometer, a portion of the light ray entering at an angle $\theta$ to the axis normal to the etalon will also leave the etalon, at the same angle $\theta$, as shown in Fig. 4. Because the etalon mirrors are partially reflecting, a portion of the light ray will also be reflected two times and will then leave the etalon parallel to the first transmitted ray. This pattern of multiple reflections will be repeated and as shown, will lead to multiple transmitted beams. All the rays that are parallel, and in the same plane of incidence, will combine at a point P on the screen. Since the individual rays are not coherent with each other, the intensity at P will simply be the sum of the intensities of the individual waves. The resulting interference pattern is a series of concentric light and dark rings.

The fringe system of a Fabry-Perot Interferometer is the same as the basic equation for the cavity modes in the resonator, but is generalized to include light rays at an angle $q$ to the normal. The path of the ray is resolved into components parallel and perpendicular to the normal at the mirror face, so that the parallel component (which contributes to the fringe intensity) is given by $k \cos(\theta)$ (with $k = 2\pi/\lambda$). The resulting equation is:

$$m \lambda = 2 n L \cos(\theta)$$

where $m$ is the fringe index (“fringe order”)
- $\lambda$ is the free-space wavelength of light used
- $n$ is the refractive index of the material inside the etalon (=1 in our case but may be very different from 1 for laser cavities filled with an active medium, for example)
- $L$ is the separation between the etalon mirror surfaces inside the cavity

**Experimental Procedure: Fabry-Perot**

Measuring the Sodium Doublet Separation on the Fabry-Perot Interferometer:

The length $L$ of the Fabry-Perot interferometer is adjusted by using a micrometer screw to move one of the parallel mirrors forming the etalon. The mirror position can be read on the micrometer, which is calibrated in millimeters. The mirror is moved by a lever connected to the micrometer screw, so the ratio of the micrometer reading to the actual movement of the mirror is 1:5.

Then, with air for the medium between the mirrors, we have $n = 1$ and, at the center of the fringe pattern, $\cos(\theta) = 1$. The fringe system equation becomes:

$$m \lambda = 2 L$$

As noted above, the resolving power of a Fabry-Perot etalon is extremely high. Thus it is well-suited to be used to measure closely space wavelengths such as the Na doublet presents. Using our Na light source, a set of two superimposed fringe patterns from the Na doublet can be observed. The Na doublet consists of two spectral lines in the yellow having wavelengths of 5890 and 5896 Angstroms. In a Na

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discharge lamp these two wavelengths are emitted incoherently and thus each will present its own fringe pattern, which our eye will see as being superimposed one on top of the other. We can distinguish the patterns due to the two wavelengths by their intensities; the 5890 Å spectral emission line is twice as intense as the 5896 Å spectral emission line (this has to do with the so-called “oscillator strengths” and “transition matrix elements” of the quantum states of the outer shell electron in the sodium atom, which would be covered in an atomic physics course). Because of the differing intensities of the two emission lines, the movable mirror can be adjusted so that the ultrafine fringes due to the weaker 5896 Å line will appear to be exactly halfway between the heavier fringes due to the 5890 Å line. **Adjust the Fabry Perot interferometer mirror spacing using the micrometer to achieve this condition. Record the micrometer setting at which you achieve this.**

Now our task is to measure the two separate wavelengths (call them $\lambda_1$ and $\lambda_2$) of the sodium doublet. This is colloquially referred to as “resolving the doublet”.

The first micrometer reading taken above corresponds to a mirror spacing $L=L_1$ such that:

$$2 L_1 = m_1 \lambda_1 = (m_2 + p + 1/2) \lambda_2$$

where $\lambda_1$ is greater than $\lambda_2$ (i.e. we choose $\lambda_1$ to represent the longer of the two wavelengths—we can do this without loss of generality). The factor of $1/2$ in the last term on the right-hand side means that the fringe order of the shorter wavelength ring system differs from that of the longer wavelength ring system by an odd half integer. This is by design: remember we adjusted the etalon mirror spacing so that the ring patterns have been adjusted to fall midway between each other, with the dark part of one wavelength’s fringe pattern overlaying the bright part of the other wavelength’s pattern.

The mirror is then adjusted, and the fringe pattern will seem to move outwards from the center of the pattern. When the fine rings are once again halfway between the heavier rings, a second reading of the micrometer is taken to determine the new mirror spacing $L=L_2$ is taken:

$$2 L_2 = m_2 \lambda_1 = (m_2 + p + 3/2) \lambda_2$$

(Note that in general we do not start with the plates in contact, i.e. we never have the condition $L=0$. In fact we cannot since the physical contact would damage the delicate partially reflecting surfaces. The integer $p$ is introduced in the above two equations to account for the non-zero mirror starting separation.)

Subtracting these two equations gives us:

$$2 (L_2 - L_1) = (m_2 - m_1) \lambda_1 = (m_2 - m_1 + 1) \lambda_2$$

$$(m_2 - m_1) (\lambda_1 - \lambda_2) = \lambda_2$$

$$\lambda_1 - \lambda_2) = \lambda_2/[2 (L_2 - L_1)]$$

Since $\lambda_1$ and $\lambda_2$ are so close for the sodium doublet, we may take them to be approximately equal to the average wavelength Na yellow light wavelength $\lambda$, i.e. $\lambda_1 \approx \lambda_2 \approx \lambda$. This average wavelength may be determined fringe contrast counting for either ring pattern separately, or it may bet determined separately using the Michelson interferometer, for example. Writing our above result in terms of the average wavelength $\lambda$ in the numerator we get:

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\((\lambda_1 - \lambda_2) = \lambda^2 / [2 \ (L_2 - L_1)]\)

This expresses our desired doublet wavelength separation in terms of the measured mirrors position \(L_1\) and \(L_2\). The mirror separation \((L_2 - L_1)\) is evaluated from micrometer readings as:

\((L_2 - L_1) = 0.10 \ (D_2 - D_1) \ K\),

where \((D_2 - D_1)\) is the change of the micrometer reading as read in millimeters, and \(K\) is the ratio of the mirror carriage movement to micrometer reading; for our system \(K = 0.20\) because there is a lever arm with a 5:1 lever ratio connecting the micrometer lead screw to the moveable mirror carriage.

Using the above calculate the separation of the Na doublet spectral lines. Express your final result in Ångstroms. Compare the values to the accepted value of 6 Angstroms (\(D1 = 5896\) A, \(D2 = 5890\) A). How close is your result to the expected value?

**PART 3:** Shearing Interferometer: Set up the

**PART 4:** Holographic Interferometer

Follow the instructions in the holographic interferometer manual (separate manual). Satisfy yourself that you can see fringe shifts when the aluminum block is subjected to stress and undergoes strain.

**APPENDIX:** Theory of Fringe Contrast in the Michelson Interferometer

The sodium doublet consists of two spectral lines in the yellow having wavelengths of 5890 and 5896 Ångstroms. The 5890 A line is twice as intense as the 5896 A line. Therefore we have to consider the interference pattern when the incident light consists of wavetrains with two different but closely separated wavelengths. We can write the electric field of the incident light beam as:

\[ E_0 = A_1 \sin [kA x - g(t)] + A_2 \sin [kB x - h(t)] \]

In the incoming light, there are two wavelengths, \(\lambda_A\) and \(\lambda_B\), with electric field amplitudes amplitudes \(A\) and \(B\) respectively. (In the following discussion it will be assumed that the amplitudes of these two waves are approximately equal, i.e. that \(A_1 \approx A_2\).) It is very important to realize that the time terms \(g(t)\) and \(h(t)\) are random with respect to one another. These wavelengths arise when an outer electron of an atom undergoes a quantum jumps from a higher energy state to a lower state. The quantum states involved in the emission of light with wavelength \(\lambda_1\) are different from those involved in the emission of light with wavelength \(\lambda_2\) and the electron transitions between the first two levels are independent of the transitions between the other two levels because the jumps occur in different atoms. Hence, there is no fixed relationship in time between the appearance of the two waves, i.e. they are random in time with respect to each other. When this is the case we way the wavetrains are incoherent. Because of the incoherence of the optical emission at two wavelengths \(\lambda_1\) and \(\lambda_2\), their behavior in the Michelson interferometer must be treated individually (i.e. there can be no "interference" between the two waves of different wavelength; only wavetrains which are coherent with each other can interfere).

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Thus, each wavelength has its own intensity pattern in the interferometer, as described in the “Theory” section for the Michelson interferometer above.

The two intensity patterns in the interferometer, arising from the two wavelengths, can be represented as a function of mirror movement by the following bar diagram:

![Figure 5: Fringes for 2 wavelengths](image)

The two patterns coincide for \( l_1 - l_2 = d = 0 \), cancel each other as the mirror is moved from zero path difference, and coincide again as the mirror is moved further. This coincidence and cancellation continues as the mirror is moved. The overall fringe “visibility” will thus vary with mirror position. When the patterns for the two wavelengths coincide the fringes will be very distinct (minima black, maxima bright). When the patterns are out of step such that the maxima for one wavelength fall on the minima for the other wavelength, then fringe visibility will be poor and the individual fringes will fade out into the background because the fringe patterns for the two different wavelengths tend to cancel. This variation is shown in the diagram below.
Recall that the distance the moveable mirror M1 must be moved between consecutive fringe maxima is $\lambda/2$. Also, note from the previous bar diagram, that if the number of fringe maxima between coincidences of the two intensity patterns is $N$ for $\lambda_1$, then it is $N+1$ for $\lambda_2$.

Let $d_c$ be the distance the mirror M1 must be moved between consecutive positions of pattern coincidence (i.e. between consecutive high contrast/high visibility fringe patterns).

\[
d_c = (N+1) \left( \frac{\lambda_1}{2} \right) = N \left( \frac{\lambda_2}{2} \right)
\]
\[
N \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = N \left( \frac{\lambda_1}{2} \right)
\]
\[
\lambda_2 = N \left( \frac{\lambda_1}{2} \right)
\]
\[\text{Let } \Delta \lambda = \lambda_1 - \lambda_2 \]
\[\text{and } \lambda_{\text{avg}} = N \Delta \lambda \]
\[
N = \frac{\lambda_{\text{avg}}}{\Delta \lambda}
\]

Therefore,
\[
d_c = N \left( \frac{\lambda_1}{2} \right) \approx \left( \frac{\lambda_{\text{avg}}}{\Delta \lambda} \right) \left( \frac{\lambda_{\text{avg}}}{2} \right)
\]
\[
d_c \approx \left( \frac{\lambda_{\text{avg}}^2}{2 \Delta \lambda} \right)
\]

where $d_c$ is the distance the mirror moves (Recall also that for our interferometer the ratio of mirror : micrometer movement = 1 : 5 because of the lever arm which transmits the micrometer motion to the mirror carriage).

Note also that the result is approximate because we approximated $\lambda_1$ by $\lambda_{\text{avg}} = 5893$ Angstroms. However the degree of approximation is very high for the sodium doublet because the spectral line wavelengths are so close together (only 6 Å apart).

Finally, the doublet separation is given by $\Delta \lambda \approx \left( \frac{\lambda_{\text{avg}}^2}{2d_c} \right)$

The value of $d_c$ can be obtained from the change of the micrometer reading as read in millimetres, remember to account for the 5:1 ratio imposed by the lever arm. The overall optical intensity pattern

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observed when using the interferometer to analyze a doublet is shown schematically in the following diagram:

![Figure 7: Michelson Interferometer Fringe Visibility variation when analyzing a doublet](image)

By determining the mirror movement between the individual fringes, the average wavelength can be calculated (this was explained in detail in the Michelson Interferometer Theory section). By determining the mirror movement between two successive fringe visibility maxima (positions of coincidence) the wavelength difference between the two wavelengths can be calculated. Note that the wavelength difference can also be obtained from the mirror movement between two successive visibility minima (positions of cancellation where the individual fringes disappear into the background light) since

\[ d_c = d \text{ (coincidence)} = d \text{ (cancellation)}. \]

References